

ALGEBRA 2



PDF

RADICAL FUNCTIONS

NOTES • HOMEWORK • QUIZZES • TEST

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Unit 6 – Radical Functions: Sample Unit Outline

	TOPIC	HOMEWORK
DAY 1	n^{th} Roots & Simplifying Radicals	HW #1
DAY 2	Rational Exponents, Converting Between Rational and Radical Form	HW #2
DAY 3	Quiz 6-1	None
DAY 4	Add, Subtract, and Multiply Radicals	HW #3
DAY 5	Dividing Radicals (includes rationalizing with monomial and binomial denominators)	HW #4
DAY 6	Radicals & Rational Exponents Review	HW #5
DAY 7	Quiz 6-2 (Radicals and Rational Exponents)	None
DAY 8	Solving Radical Equations	HW #6
DAY 9	Graphing Square Root Functions	HW #7
DAY 10	Graphing Cube Root Functions	↓
DAY 11	Quiz 6-3	None
DAY 12	Inverse Functions	HW #8
DAY 13	Unit Review	Study for Test
DAY 14	Unit 6 Test	None
ADDITIONAL RESOURCES: Video Lessons Google Slides Suggested Activities		

Name:

Date:

Topic:

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Main Ideas/Questions

Notes/Examples

WARM-UP

List the perfect squares, cubes, and fourths.

Perfect Squares:

Perfect Cubes:

Perfect Fourths:

NTH ROOTS

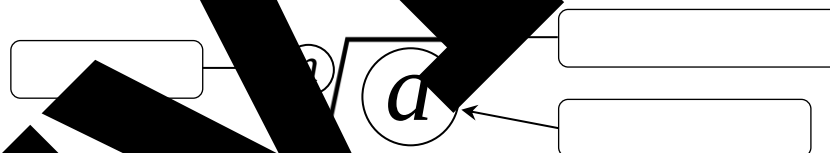
Definition: x is the n^{th} root of a real number a if $x^n = a$.

Examples:

- _____ and _____ are **square roots** of 81 because _____ and _____.
- _____ is the **cube root** of -8 because _____.
- _____ and _____ are **fourth roots** of 256 because _____ and _____.

RADICAL Expression

The n^{th} root of a real number, a , can be written as the radical expression $\sqrt[n]{a}$.



- If there is **no index**, it is assumed that _____.

Number Roots:

Index	Radicand	Type of Roots	# of Roots
Even	Positive		
Odd	Positive		
Odd	Negative		
★ Even	Negative		

If a radicand has more than one n^{th} root, the radical sign indicates only the **principal, or positive, root**.

EVALUATING Radicals

Find each value.

$$\sqrt{16} =$$

$$-\sqrt{121} =$$

$$\sqrt{289} =$$

$$-\sqrt{\frac{4}{25}} =$$

$$\sqrt[3]{8} =$$

$$\sqrt[3]{343} =$$

$$\sqrt[3]{-125} =$$

$$\sqrt[3]{-\frac{1}{27}} =$$

$$-\sqrt[4]{1} =$$

$$\sqrt[4]{2,401} =$$

$$-\sqrt[4]{4,096} =$$

$$\sqrt[4]{\frac{81}{16}} =$$

SIMPLIFYING

Radicals

1. $\sqrt{117}$

2. $4\sqrt{320}$

3. $2\sqrt[3]{48}$

4. $3\sqrt[3]{108}$

5. $\sqrt[3]{-250}$

6. $6\sqrt[3]{2}$

7. $3\sqrt[4]{162}$

8. $\sqrt[4]{32}$

Radicals with

VARIABLES

Square Roots

Exponents must be multiples of ____!

Cube Roots

Exponents must be multiples of ____!

4th Roots

Exponents must be multiples of ____!

9. $\sqrt{xy^9}$

10. $\sqrt{324a^3b^7}$

11. $\sqrt[3]{216m^6n^9}$

12. $\sqrt[3]{56r^8s^4}$

13. $\sqrt[3]{-64x^{10}y^{21}}$

14. $\sqrt[3]{-81p^2q^{12}}$

15. $\sqrt[4]{w^4v^{17}}$

16. $\sqrt[4]{48m^8n^3}$



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Homework 1: n^{th} Roots; Simplifying Radicals

For each value, give the number of possible roots, then list the roots.

1. 196, square root(s)

2. 216, cube root(s)

3. 625, fourth root(s)

Evaluate each expression

4. $\sqrt{225}$ 5. $\sqrt[3]{512}$ 6. $\sqrt[3]{-64}$ 7. $\sqrt[4]{1,296}$

Simplify each radical expression.

8. $5\sqrt{150}$ 9. $-8\sqrt{63}$ 10. $\sqrt[3]{875}$ 11. $2\sqrt[3]{-54}$ 12. $-3\sqrt[4]{56}$ 13. $6\sqrt[4]{405}$ 14. $3\sqrt{252k^5}$ 15. $4\sqrt{60a^6b}$ 16. $3\sqrt{49x^9y^{16}}$ 17. $3\sqrt[3]{82p^6q^9}$ 18. $\sqrt[3]{-r^{14}s^9}$ 19. $\sqrt[3]{-625m^{28}n^{12}}$ 20. $\sqrt[3]{27n^{13}}$ 21. $-2\sqrt[4]{80x^8y^9z}$ 22. $5\sqrt[4]{112c^{24}d^{13}}$

Name:

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Main Ideas/Questions	Notes/Examples		
RATIONAL EXPONENTS	Expressions with rational exponents can be rewritten as radicals using the following rules:		
	Exponential Form	Meaning	Radical Form
	$a^{\frac{1}{n}}$	The n^{th} root of a	$\sqrt[n]{a}$
	$a^{\frac{m}{n}}$	The n^{th} root of a , raised to the m^{th} power	$\sqrt[n]{a^m}$
<i>Converting</i> EXPONENTIAL TO RADICAL FORM	Directions: Write each expression in radical form . Simplify if needed.		
	1. $x^{\frac{1}{4}}$	2. $24^{\frac{1}{3}}$	3. $(15x)^{\frac{1}{2}}$
	4. $7^{\frac{2}{3}}$	$k^{\frac{7}{2}}$	6. $3^{\frac{5}{4}}$
	$5^{\frac{3}{4}}$	8. $(-6x)^{\frac{2}{3}}$	9. $7(12w)^{\frac{1}{2}}$
<i>Converting</i> EXPONENTIAL TO RADICAL FORM	Directions: Write each expression in exponential form .		
		11. $\sqrt{xy}$	12. $\sqrt[4]{8w^2}$
	13. $\sqrt[3]{11^2}$	14. $\sqrt[4]{k^{10}}$	15. $(\sqrt{3m})^7$
	16. $(\sqrt[4]{-2a})^5$	17. $\sqrt{10^5 a^3 b^8}$	18. $\sqrt[8]{9x^2 y^{12}}$

SIMPLIFYING EXPRESSIONS

with Rational
Exponents

- ① Rewrite all radicals in **exponential form**.
- ② Use the **exponent rules** to simplify the expression.
- ③ Write your **answer as a radical in simplest form**. Simplify if needed.

19. $x^{\frac{1}{3}} \cdot x^{\frac{4}{3}}$

20. $p^{\frac{1}{4}} \cdot p^{\frac{3}{2}}$

21. $\frac{m^{\frac{5}{2}}}{m^{\frac{7}{4}}}$

22. $\left(a^{\frac{1}{3}}\right)^{\frac{5}{2}}$

23. $\left(32^{\frac{1}{2}}\right)^{\frac{1}{2}}$

25. $100^{-\frac{1}{2}}$

$\cdot 16^{\frac{1}{3}}$

27. $(216)^{-\frac{1}{3}}$

28. $\left(\frac{112}{7}\right)^{-\frac{1}{4}}$

29. $\sqrt[3]{v} \cdot \sqrt{v}$

30. $\sqrt[4]{r^3} \cdot \sqrt{r}$

31. $\frac{4}{\sqrt[3]{4}}$

32. $\frac{\sqrt{7^3}}{\sqrt{7}}$

33. $\sqrt[4]{x^{10}}$

34. $\sqrt[4]{25m^2}$



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Homework 2: Rational Exponents

Directions: Rewrite each expression in **radical form**. Simplify if needed.

1. $28^{\frac{1}{2}}$

2. $2^{\frac{4}{3}}$

3. $x^{\frac{5}{4}}$

4. $(256x)^{\frac{1}{4}}$

5. $(mn)^{\frac{7}{2}}$

6. $(x^4y^3)^{\frac{1}{2}}$

Directions: Rewrite each expression in **exponential form**.

7. $\sqrt[4]{10^3}$

8. $\sqrt{3ab}$

9. $(-2x^5)^5$

10. $\sqrt[3]{x^6y^2}$

Directions: Simplify each expression. Give answers in **radical form**.

11. $9^{\frac{1}{2}} \cdot 9^{\frac{5}{2}}$

12. $x^{\frac{7}{3}}$

13. $(28^{\frac{3}{5}})^{\frac{5}{6}}$

14. $(-64)^{-\frac{1}{3}}$

15. $45^{-\frac{3}{2}}$

16. $2\left(\frac{48}{3}\right)^{-\frac{1}{4}}$

17. $\sqrt[3]{16}$

18. $\frac{\sqrt[3]{24^4}}{24}$

19. $\frac{m}{\sqrt[4]{m}}$

20. $\frac{\sqrt[4]{16}}{\sqrt[4]{16^3}}$

21. $\sqrt[4]{a^2b^{14}}$

22. $\sqrt[4]{36w^6}$

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Unit 6: Radical Functions

Quiz 6-1: Simplifying Radicals & Rational Exponents

Write each expression in simplest radical form.

1. $\sqrt{245}$

2. $6\sqrt[3]{-72}$

3. $-\sqrt[4]{486}$

4. $\sqrt[4]{144m^7}$

5. $\sqrt{600p^9q^{16}}$

6. $\sqrt[3]{250}$

4. _____

Write each expression in exponential form.

7. $\sqrt[3]{14x^7y^4}$

8. $\sqrt[4]{2c^2d^6}$

6. _____

7. _____

8. _____

9. _____

10. _____

11. _____

12. _____

13. _____

14. _____

Simplify. Write each expression in simplest radical form.

9. $4^3 \cdot 4^{-\frac{5}{3}}$

10. $\left(\frac{5}{8w^8}\right)^4$

12. $(c^2)^{\frac{7}{4}} \cdot (c^2)^{\frac{9}{4}}$

11. $k^{-\frac{2}{3}}$

13. $(\sqrt{r} \cdot \sqrt[4]{r^5})^7$

14. $\frac{\sqrt[3]{y^{17}}}{y^2}$

Name:

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Main Ideas/Questions

Notes/Examples

ADDING & SUBTRACTING

Radicals

①

SIMPLIFY all radicals.

②

Identify radicals with the **SAME INDEX** and **SAME RADICAND**. Only these can be combined!

③

For common radicals, **add/subtract the coefficients** and **KEEP THE COMMON RADICAL**.

1. $3\sqrt{27} - 2\sqrt{12}$

2. $3\sqrt{54} - 2\sqrt{18}$

3. $7\sqrt[4]{48} - 2\sqrt[4]{3} + 3\sqrt[3]{72}$

4. $10\sqrt{20} + 5\sqrt{56} - 4\sqrt{175}$

5. $4x^2y^2 - 3x^2y\sqrt{2}$

6. $\sqrt[3]{-40a^7} + 2a^2 \cdot \sqrt[3]{135a^4}$

MULTIPLYING

Radicals

②

SIMPLIFY the resulting radical.

7. $\sqrt{27} \cdot \sqrt{5}$

8. $3\sqrt{10} \cdot -2\sqrt{18}$

9. $2\sqrt[3]{9} \cdot 5\sqrt[3]{-24}$

10. $-3\sqrt[4]{64} \cdot -\sqrt[4]{8}$

$$11. \sqrt{6x^4} \cdot 5\sqrt{8x^5}$$

$$12. \sqrt[3]{54m^8} \cdot \sqrt[3]{5m^4}$$

$$13. \sqrt[3]{-3a^7b^4} \cdot \sqrt[3]{36a^6b^2}$$

$$14. 2\sqrt[4]{p^2q} \cdot 7\sqrt[4]{p^3}$$

BINOMIAL

Examples

$$15. \sqrt{10}(5\sqrt{5} - 2\sqrt{2})$$

$$16. (8\sqrt{3} - 5)(3 - \sqrt{10})$$

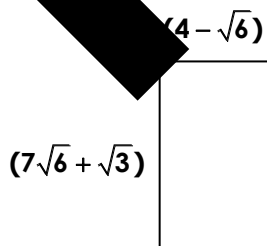
$$17. (6\sqrt{6} - \sqrt{5} + \sqrt{2})(\sqrt{3} - \sqrt{2})$$

$$18. (4 - \sqrt{5})^2$$

$$19. \sqrt{3k}(\sqrt{15k} - 2\sqrt{15k^2})$$

$$20. (\sqrt{x} - \sqrt{8})(\sqrt{x} + \sqrt{2})$$

21. Find the area and perimeter of the rectangle shown to the left.



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Homework 3: Add/Subtract/Multiply Radicals

Directions: Simplify each expression below.

1. $7\sqrt[3]{3} + 2\sqrt[3]{192}$

2. $10\sqrt{7} - \sqrt{28} + 6\sqrt{180}$

3. $8\sqrt[4]{48} - 5\sqrt[4]{90} + 9\sqrt[4]{3}$

4. $5\sqrt[3]{32x^3y^4} - 2\sqrt[3]{4y}$

5. $15\sqrt{2} \cdot -2\sqrt{20}$

6. $4\sqrt{12} \cdot \sqrt{48}$

7. $3\sqrt{18w^7} \cdot 10\sqrt{4w^9}$

8. $\sqrt{8a^3b} \cdot \sqrt[4]{10a^2b^7}$

9. $2\sqrt{3}(\sqrt{27})$

10. $(5\sqrt{5} - 2)(\sqrt{5} - 1)$

11. $(1 + \sqrt{7})(10 - 3\sqrt{7})$

12. $(3 - 2\sqrt{k})^2$

Name:

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Main Ideas/Questions	Notes/Examples	
DIVIDING <i>Radicals</i>	①	Divide coefficients, then use the QUOTIENT RULE : $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$
	②	SIMPLIFY the resulting radical.
	1. $\frac{12\sqrt{160}}{2\sqrt{5}}$	2. $\frac{36\sqrt[4]{1,250}}{9\sqrt[4]{2}}$
	3. $\frac{\sqrt{x^3y^9}}{\sqrt{x^2y^5}}$	4. $\frac{\sqrt{48m^6}}{\sqrt{3}}$
	5. $\sqrt{\frac{48}{1}}$	6. $\sqrt[3]{\frac{40}{27}}$
	7. $\frac{\sqrt[3]{7x^5}}{\sqrt[4]{64y^6}}$	8. $\sqrt[4]{\frac{32w}{81}}$
RATIONALIZING <i>the denominator</i>	* Monomial Denominators: Multiply the numerator and denominator by the radical. * Binomial Denominators: Multiply the numerator and denominator by the conjugate. (The same expression but with the opposite sign in the middle.)	
	9. $\frac{3}{2\sqrt{5}}$	10. $\sqrt{\frac{8}{15}}$

$$11. \frac{3\sqrt{12}}{4\sqrt{7}}$$

$$12. \frac{\sqrt{32a^5}}{\sqrt{3a}}$$

$$13. \frac{\sqrt{8} - \sqrt{2}}{\sqrt{2}}$$

$$14. \frac{3\sqrt{2} + \sqrt{5}}{\sqrt{12}}$$

$$15. \frac{4}{4 + \sqrt{2}}$$

$$16. \frac{2}{4 - \sqrt{5}}$$

$$17. \frac{1}{1 - 4\sqrt{3}}$$

$$18. \frac{5 - \sqrt{5}}{\sqrt{5} + \sqrt{3}}$$

$$19. \frac{3 + \sqrt{7}}{2 + 2\sqrt{7}}$$

$$20. \frac{8 - \sqrt{6}}{3 - 4\sqrt{6}}$$

BINOMIAL
Denominators



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Homework 4: Dividing Radicals

**** This is a 2-page document! ****

Directions: Simplify each expression below.

1. $\frac{20\sqrt{270}}{4\sqrt{3}}$

2. $\frac{6\sqrt[4]{336}}{2\sqrt[4]{7}}$

3. $\frac{\sqrt[3]{a^{10}b^{16}}}{\sqrt[3]{a^5b^4}}$

4. $\frac{24\sqrt{22}}{\sqrt{5x^4}}$

5. $\sqrt[3]{\frac{-324}{2}}$

$\frac{2m^7}{625n^4}$

7. $\sqrt{\frac{3}{20}}$

8. $\frac{\sqrt{8}}{2\sqrt{14}}$

9. $\frac{1}{8\sqrt{2}}$

10. $\sqrt{\frac{18}{30}}$

$$11. \frac{\sqrt{5n}}{2\sqrt{15n^4}}$$

$$12. \frac{4y}{\sqrt{2x^4y}}$$

$$13. \frac{\sqrt{5}-\sqrt{6}}{\sqrt{18}}$$

$$14. \frac{4-\sqrt{2}}{2\sqrt{8}}$$

$$15. \frac{12}{4-\sqrt{3}}$$

$$\frac{\sqrt{2}}{\sqrt{2}}$$

$$17. \frac{2\sqrt{6}}{6}$$

$$18. \frac{6+5\sqrt{2}}{4+3\sqrt{2}}$$

REVIEW: Radicals & Rational Exponents

SIMPLIFYING RADICALS

Write each expression in simplest radical form.

1. $\sqrt{300m^4n^{25}}$

2. $\sqrt[3]{-72a^9b^2}$

3. $\sqrt[4]{81m^{11}n^{20}}$

OPERATIONS WITH RADICALS

Perform the indicated operation(s). Write each answer in simplest radical form.

4. $4\sqrt{98} + \sqrt{150} - 2\sqrt{32}$

5. $10\sqrt[4]{5} + 7\sqrt[4]{2} - \sqrt[3]{320}$

6. $3\sqrt[4]{8} \cdot 7\sqrt[4]{32}$

7. $\sqrt[3]{-6x^3} \cdot \sqrt[4]{-4x^4}$

8. $\sqrt{3}(2 - \sqrt{24})$

9. $(4 + 3\sqrt{5}) \cdot (-2 + \sqrt{5})$

10. $(1 - 7\sqrt{3})(1 + 7\sqrt{3})$

11. $(3 + \sqrt{6})^2$

12. $\frac{-45}{9\sqrt{3}}$

13. $\frac{\sqrt[4]{r^{20}s^9}}{\sqrt[4]{r^3s}}$

14. $\sqrt{\frac{50}{81}}$	15. $\sqrt[4]{\frac{3a}{16b^8}}$
16. $\frac{\sqrt{6}}{4\sqrt{3}}$	17. $\sqrt{\frac{8x}{5}}$
18. $\frac{2-\sqrt{12}}{\sqrt{3}}$	19. $\frac{3+\sqrt{3}}{4\sqrt{12}}$
20. $\frac{5-4\sqrt{2}}{4-3\sqrt{2}}$	21. $\frac{3\sqrt{3}}{4\sqrt{12}}$

RATIONAL EXPONENTS

Simplify each expression. Write the final answer in simplest radical form.

22. $x^{\frac{2}{3}} \cdot x^{\frac{11}{3}}$	23. $32^{\frac{3}{8}}$	24. $n^{\frac{7}{3}} \cdot n^3$
25. $\frac{7^4}{\frac{1}{5}}$	26. $\left(k^{\frac{3}{2}}\right)^{\frac{1}{2}}$	27. $1,296^{\frac{1}{4}}$
28. $\frac{125^{\frac{1}{3}}}{125^{\frac{2}{3}}}$	29. $\frac{\sqrt[4]{3^3}}{\sqrt{3}}$	30. $\sqrt[4]{10^2 a^{18}}$

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Unit 6: Radical Functions

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Homework 5: Radicals & Rational Exponents

** This is a 2-page document! **

Directions: Simplify each expression.

1. $7\sqrt{192m^9n^2}$

2. $\sqrt[3]{-135x^6y^{13}}$

3. $\sqrt[4]{1,250a^7b^{24}}$

4. $-2\sqrt{27} - \sqrt{18} + 3\sqrt{72}$

5. $2\sqrt[4]{405} - 3\sqrt[4]{125} + \sqrt{192}$

6. $-3\sqrt{6} \cdot -6\sqrt{8}$

7. $5\sqrt[3]{-4} + \sqrt[3]{36} - \sqrt[3]{3}$

8. $\sqrt{15}(2\sqrt{6} - 7\sqrt{5})$

9. $(3\sqrt{3} - 2)(4\sqrt{3} + 5)$

10. $(1 - 7\sqrt{2})(1 + 7\sqrt{2})$

11. $(5 + \sqrt{12})^2$

12. $\frac{-2\sqrt{3}}{5\sqrt{6}}$

13. $\sqrt{\frac{12}{27}}$

14. $\frac{8\sqrt[3]{144a^{15}}}{4\sqrt[3]{3a^5}}$	15. $\frac{5\sqrt[4]{2}}{4\sqrt[4]{162}}$	
16. $\frac{6\sqrt{20}}{\sqrt{18}}$	17. $\frac{4+\sqrt{6}}{3\sqrt{6}}$	
18. $\frac{7}{4+\sqrt{5}}$	19. $\frac{5\sqrt{2}}{-\sqrt{2}}$	
Directions: Use the exponent rules to simplify each expression. Write your answers in simplest radical form.		
20. $5r^3 \cdot 2r^{\frac{1}{2}}$	21. $(3v)^{\frac{2}{3}} \cdot (3v)$	22. $\frac{n^{\frac{7}{4}}}{\frac{5}{n^4}}$
23. $(243x^5)^{\frac{1}{5}}$	24. $\frac{\left(54^{\frac{1}{2}}\right)^{\frac{5}{3}}}{54^{\frac{1}{2}}}$	25. $\left(\frac{144}{9}\right)^{-\frac{1}{4}}$
26. $4p^{\frac{5}{6}}$	27. $\frac{m^2}{\sqrt[3]{m}}$	28. $\sqrt[4]{49y^{22}}$

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Unit 6: Radical Functions

Quiz 6-2: Radicals (Simplifying & Operations), Rational Exponents

Part I: Radicals – Simplify. Write all final answers in simplest radical form.

1. $\sqrt[3]{256a^{15}b^2}$

2. $8\sqrt[4]{32} - \sqrt[4]{162}$

1. _____

3. _____

3. $\sqrt[3]{-3x^2} \cdot \sqrt[3]{24x^5}$

4. $\sqrt{2}(\sqrt{5} - 6\sqrt{12}) + \sqrt{40}$

4. _____

6. _____

7. _____

5. $(4\sqrt{7} + 3)(6\sqrt{7} - 1)$

6. $(\sqrt{2} + \sqrt{3})^2$

8. _____

9. _____

10. _____

7. $\frac{-8\sqrt{15}}{2\sqrt{5}}$

8. $\frac{\sqrt[4]{p^7q^{10}}}{\sqrt[4]{p^2q^2}}$

9. $\frac{2}{\sqrt{12}}$

10. $\frac{10\sqrt{18}}{\sqrt{5}}$

$$11. \frac{\sqrt{8m^2}}{\sqrt{6m}}$$

$$12. \frac{\sqrt{5} + \sqrt{2}}{\sqrt{8}}$$

$$13. \frac{6}{4 + \sqrt{5}}$$

$$14. \frac{4 + \sqrt{2}}{5\sqrt{2} - 2}$$

11. _____

12. _____

13. _____

14. _____

Part II: Rational Exponents – Write final answers in radical form.

$$15. 2^{\frac{1}{2}} \cdot 2^3$$

$$16. \frac{p^{\frac{5}{3}}}{p^{\frac{1}{3}}}$$

15. _____

16. _____

$$17. (y^2)^{\frac{3}{4}}$$

$$18. \left(\frac{75}{3}\right)^{\frac{2}{3}}$$

17. _____

18. _____

$$19. 64^{\frac{2}{3}} \cdot 64$$

$$20. \frac{\sqrt{c^3}}{\sqrt[4]{c^5}}$$

19. _____

20. _____

B. _____

BONUS: $\sqrt[4]{16x^2y^6} \cdot \sqrt{28x^{10}y^{22}}$

Name:	Date:
Topic:	Class:

Main Ideas/Questions	Notes
<i>Solving</i> RADICAL EQUATIONS	① ISOLATE the radical on one side of the equation.
	② RAISE EACH SIDE OF THE EQUATION TO THE POWER OF THE INDEX to eliminate the radical sign.
	③ SOLVE the remaining equation.
	④ CHECK for extraneous solutions.

Directions: Solve each equation. Be sure to check for extraneous solutions.	
1. $\sqrt{x} + 5 = 12$	2. $3 - \sqrt[4]{m} = 2$
3. $10 + \sqrt{a+7} = 11$	4. $\sqrt{6w-5} = 3$
5. $-36 = -9(2)^2$	6. $(7p-1)^{\frac{1}{3}} + 11 = 7$
7. $\sqrt{2} - 9 = -11$	8. $-4\sqrt{43-3x} + 18 = -2$

$$9. \sqrt{u-8} = \sqrt{43-2u}$$

$$10. \sqrt[4]{27-2d} = \sqrt[4]{d-3}$$

$$11. 4\sqrt{3q+15} = 12\sqrt{q}$$

$$12. \sqrt[3]{14-10n} = -2\sqrt[3]{n}$$

$$13. \sqrt{8x} = x$$

$$14. \sqrt{14-7a} = 8a$$

$$15. \sqrt{6m-38} = m-5$$

$$16. \sqrt{2v+3} = v+2$$

$$17. \sqrt{y} = y-11$$

$$18. \sqrt{x+9} = \sqrt{x}-1$$

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Homework 6: Radical Equations



**** This is a 2-page document! ****

Directions: Solve each equation. Be sure to check for extraneous solutions.

1. $\sqrt{3-2v}+17=20$

2. $\sqrt{4k-11}+15=2$

3. $-7\sqrt[4]{m+1}=14$

4. $2(x+7)^2=18$

5. $-7+(16p+8)^{\frac{1}{3}}=-1$

6. $17-2\sqrt{3x+10}=1$

7. $\sqrt{3k+1}=\sqrt{41-2k}$

8. $(3x-21)^{\frac{1}{3}}=(19-x)^{\frac{1}{3}}$

$$9. \sqrt{19x+24} = -5\sqrt{x}$$

$$10. \frac{\sqrt[4]{7w-5}}{2} = \sqrt[4]{w-2}$$

$$11. \sqrt{63-2x} = x$$

$$12. n = (12-n)^{\frac{1}{2}}$$

$$13. \sqrt{5y-15} = y-3$$

$$\sqrt{6c+15} = c+4$$

$$15. \sqrt{4a-15} + 7 = a$$

$$16. \sqrt{q-21} = \sqrt{q} + 7$$

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Main Ideas/Questions

Notes/Examples

SQUARE ROOT

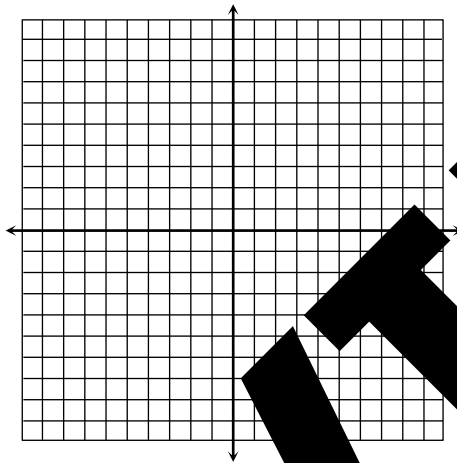
Function

Parent Function:

A **radical function** is a function of the form $f(x) = \sqrt[n]{x}$.

The **square root function** is a type of radical function.

Graph the parent function of the square root function below and identify the key characteristics.



D: _____ R: _____

End Point: _____

End Behavior: _____

As $x \rightarrow \infty$, $f(x) \rightarrow$ _____As $x \rightarrow -\infty$, $f(x) \rightarrow$ _____

Increasing Interval(s): _____

Decreasing Interval(s): _____

TRANSFORMATIONS

$$f(x) = a\sqrt{x-h} + k$$

• h is the _____ shift. (+ shifts _____, - shifts _____)

• k is the _____ shift. (+ shifts _____, - shifts _____)

• The endpoint of the function is _____.

• If a is negative, the function is _____ across the _____-_____.

• $a > 1$ represents a vertical _____.

• $0 < |a| < 1$ represents a vertical _____.

Identify the transformations of each function compared to the parent function.

1. $f(x) = \sqrt{x-7} + 2$

2. $f(x) = 4\sqrt{x} - 1$

3. $f(x) = -\sqrt{x+3}$

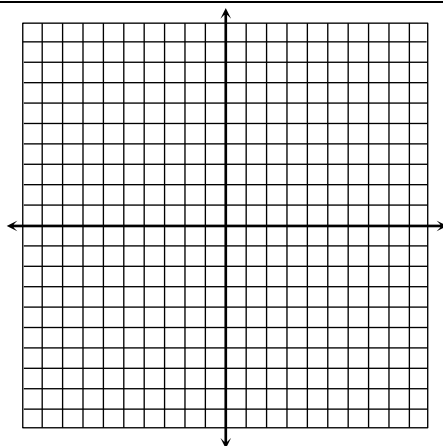
4. $f(x) = \frac{1}{2}\sqrt{x-2} - 6$

5. The square root parent function is reflected across the x -axis, then shifted 7 units left and 1 unit down. Write an equation that represents this function.

GRAPHING SQUARE ROOT FUNCTIONS

Graph each function. Identify the key characteristics.

6. $f(x) = \sqrt{x} + 5$



D: _____ R: _____

Endpoint: _____

End Behavior:

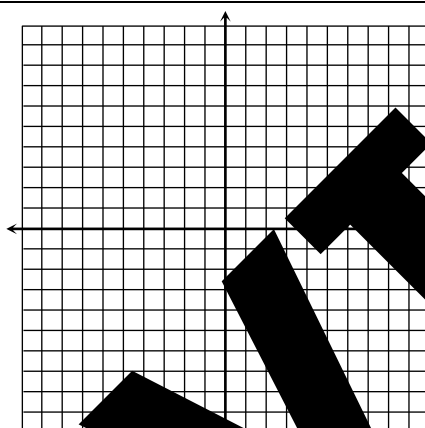
As $x \rightarrow$ _____, $f(x) \rightarrow$ _____

As $x \rightarrow$ _____, $f(x) \rightarrow$ _____

Increasing Interval(s): _____

Decreasing Interval(s): _____

7. $f(x) = \sqrt{x-1} - 4$



D: _____ R: _____

Endpoint: _____

End Behavior:

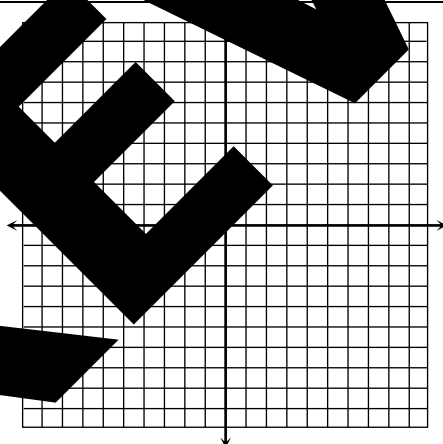
As $x \rightarrow$ _____, $f(x) \rightarrow$ _____

As $x \rightarrow$ _____, $f(x) \rightarrow$ _____

Increasing Interval(s): _____

Decreasing Interval(s): _____

8. $f(x) = 3\sqrt{x+1}$



D: _____ R: _____

Endpoint: _____

End Behavior:

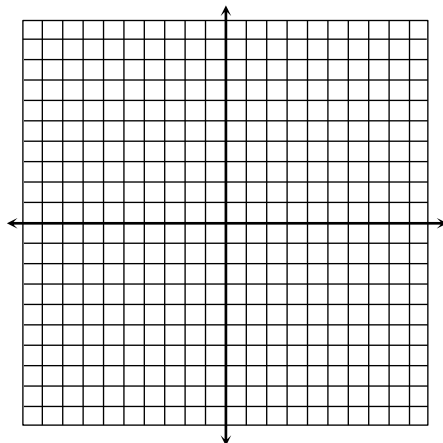
As $x \rightarrow$ _____, $f(x) \rightarrow$ _____

As $x \rightarrow$ _____, $f(x) \rightarrow$ _____

Increasing Interval(s): _____

Decreasing Interval(s): _____

$f(x) = \sqrt{x-2} + 3$



D: _____ R: _____

Endpoint: _____

End Behavior:

As $x \rightarrow$ _____, $f(x) \rightarrow$ _____

As $x \rightarrow$ _____, $f(x) \rightarrow$ _____

Increasing Interval(s): _____

Decreasing Interval(s): _____

Name:

Date:

Topic:

Class:

Main Ideas/Questions

Notes/Examples

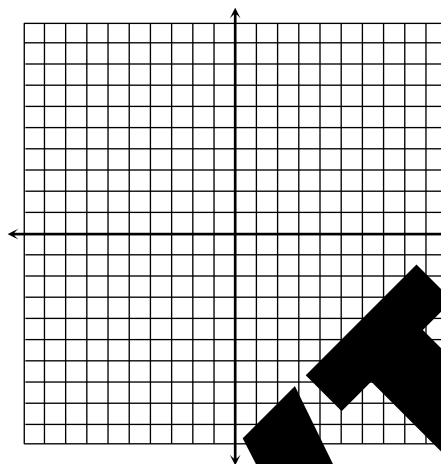
CUBE ROOT

Function

Parent Function:

The **cube root function** is another type of radical function.

Graph the parent function of the cube root function below and identify the key characteristics.



D: _____

Point of Inflection: _____

End Behavior:

As $x \rightarrow \infty$, $f(x) \rightarrow$ _____As $x \rightarrow -\infty$, $f(x) \rightarrow$ _____

Increasing interval(s): _____

Decreasing interval(s): _____

TRANSFORMATIONS

$$f(x) = a\sqrt[3]{x-h} + k$$

- h is the _____ shift. (+ shifts _____, - shifts _____)
- k is the _____ shift. (+ shifts _____, - shifts _____)
- Point of Inflection: _____
- If a is negative, the function is _____ across the _____ - _____
- If $a > 1$, represents a vertical _____.
- If $0 < a < 1$, represents a vertical _____.

Describe the transformations of each function compared to the parent function.

1. $f(x) = \sqrt[3]{x+9}$

2. $f(x) = 5\sqrt[3]{x-2}$

3. $f(x) = -\sqrt[3]{x+4} - 7$

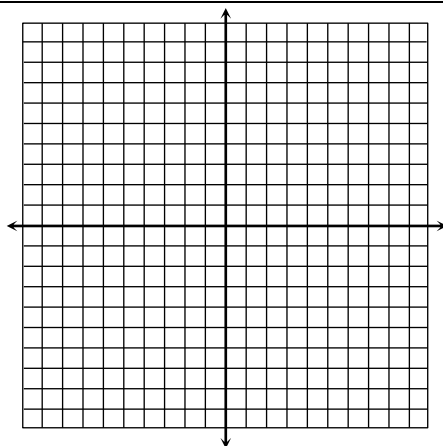
4. $f(x) = \frac{3}{4}\sqrt[3]{x-6} + 1$

5. The cube root parent function is vertically stretched by a factor of 3, then translated so that its point of inflection is located at $(-2, 3)$. Write an equation that represents this new function.

GRAPHING CUBE ROOT FUNCTIONS

Graph each function. Identify the key characteristics.

6. $f(x) = \sqrt[3]{x+3} + 2$



D: _____ R: _____

Point of Inflection: _____

End Behavior:

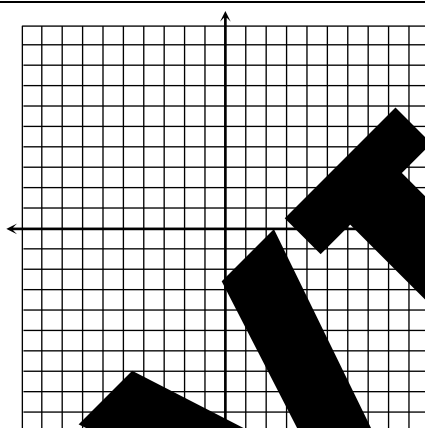
As $x \rightarrow$ _____, $f(x) \rightarrow$ _____

As $x \rightarrow$ _____, $f(x) \rightarrow$ _____

Increasing Interval(s): _____

Decreasing Interval(s): _____

7. $f(x) = -2\sqrt[3]{x} - 1$



D: _____ R: _____

Point of Inflection: _____

End Behavior:

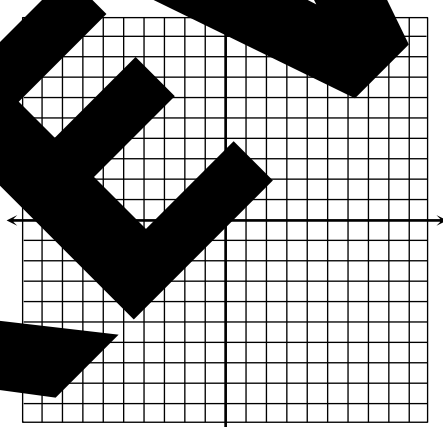
As $x \rightarrow$ _____, $f(x) \rightarrow$ _____

As $x \rightarrow$ _____, $f(x) \rightarrow$ _____

Increasing Interval(s): _____

Decreasing Interval(s): _____

8. $f(x) = \frac{1}{2}\sqrt[3]{x+1} - 5$



D: _____ R: _____

Point of Inflection: _____

End Behavior:

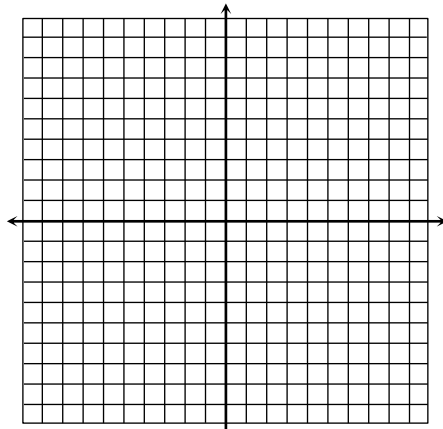
As $x \rightarrow$ _____, $f(x) \rightarrow$ _____

As $x \rightarrow$ _____, $f(x) \rightarrow$ _____

Increasing Interval(s): _____

Decreasing Interval(s): _____

9. $f(x) = \sqrt[3]{x-4} + 3$



D: _____ R: _____

Point of Inflection: _____

End Behavior:

As $x \rightarrow$ _____, $f(x) \rightarrow$ _____

As $x \rightarrow$ _____, $f(x) \rightarrow$ _____

Increasing Interval(s): _____

Decreasing Interval(s): _____

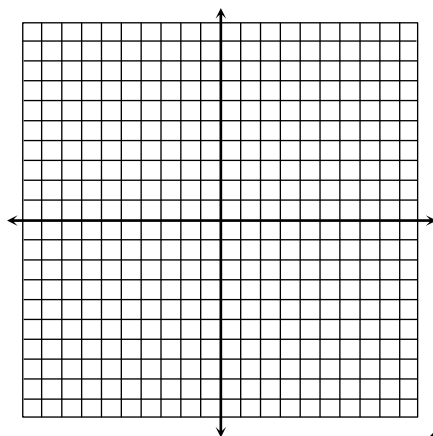
SQUARE ROOT Function

PARENT FUNCTION:

Domain: _____

Range: _____

Endpoint: _____



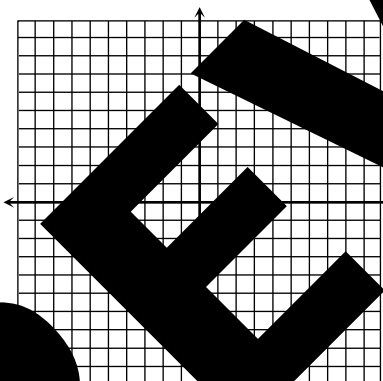
..... Transformations

EXAMPLE 1: $f(x) = \sqrt{x+1} - 3$

Domain: _____

Range: _____

Endpoint: _____

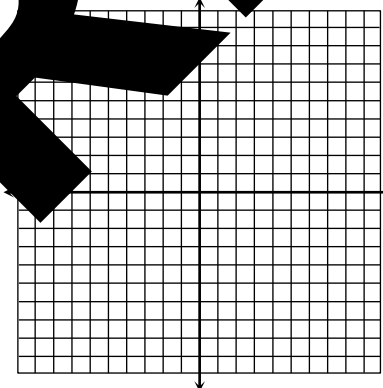


EXAMPLE 2: $f(x) = -\sqrt{x+1} - 3$

Domain: _____

Range: _____

Endpoint: _____



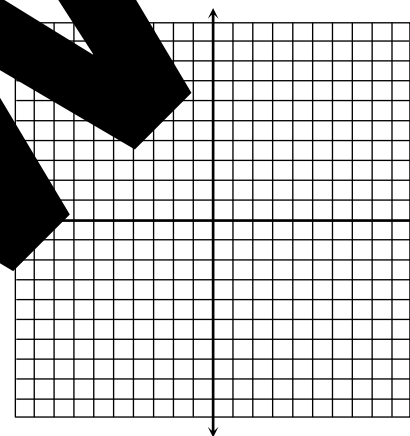
CUBE ROOT Function

PARENT FUNCTION:

Domain: _____

Range: _____

Point of Inflection: _____



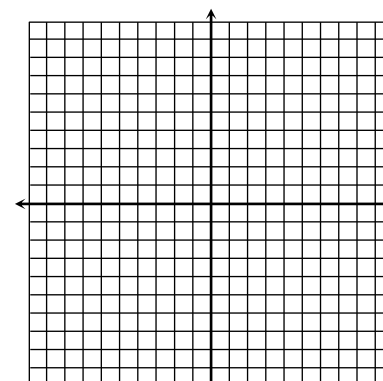
..... Transformations

EXAMPLE 1: $f(x) = \sqrt[3]{x} - 2$

Domain: _____

Range: _____

Point of Inflection: _____

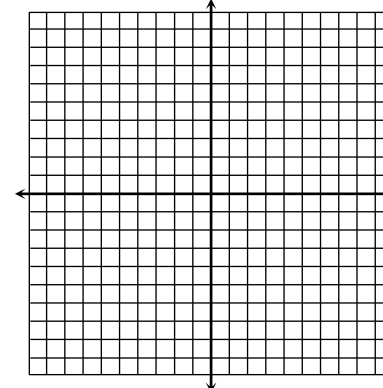


EXAMPLE 2: $f(x) = 2\sqrt[3]{x+5} + 4$

Domain: _____

Range: _____

Point of Inflection: _____



Name: _____

Unit 6: Radical Functions

Date: _____ Per: _____

Homework 7: Graphing Radical Functions

**** This is a 2-page document! ****

For questions 1-2: Describe the transformations from the parent function.

1. $f(x) = 2\sqrt{x} - 9$

2. $f(x) = -\sqrt[3]{x+5} + 3$

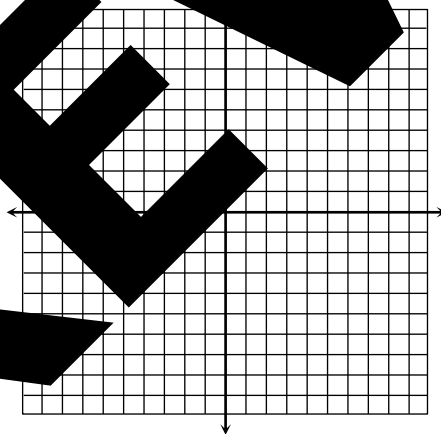
3. $f(x) = \frac{1}{4}\sqrt{x-5}$

4. The cube root parent function is vertically stretched by a factor of 3, reflected in the y-axis, then translated 8 units down. Write an equation to represent the new function.

5. The square root parent function is vertically compressed by a factor of 3, then translated so that it has an endpoint located at (4, -1). Write an equation to represent the new function.

Directions: Graph each function and label its characteristics.

6. $f(x) = \sqrt{x} - 4$



D: _____ R: _____

Endpoint: _____

End Behavior:

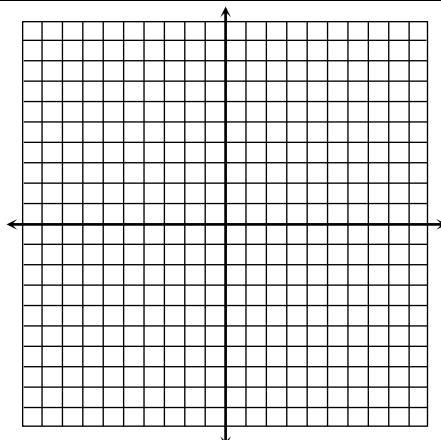
As $x \rightarrow$ _____, $f(x) \rightarrow$ _____

As $x \rightarrow$ _____, $f(x) \rightarrow$ _____

Increasing Interval(s): _____

Decreasing Interval(s): _____

7. $f(x) = \sqrt{x+2}$



D: _____ R: _____

Endpoint: _____

End Behavior:

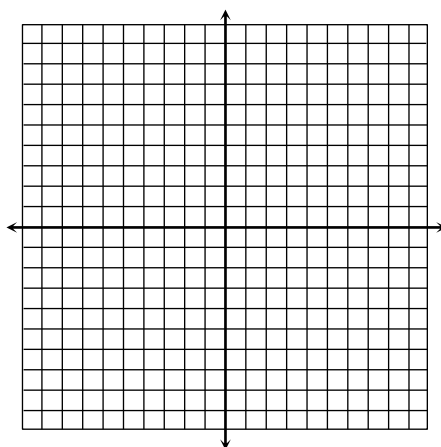
As $x \rightarrow$ _____, $f(x) \rightarrow$ _____

As $x \rightarrow$ _____, $f(x) \rightarrow$ _____

Increasing Interval(s): _____

Decreasing Interval(s): _____

8. $f(x) = 2\sqrt{x+3} + 1$



D: _____ R: _____

Endpoint: _____

End Behavior:

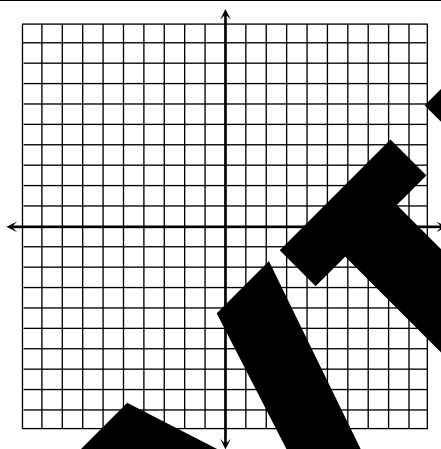
As $x \rightarrow$ _____, $f(x) \rightarrow$ _____

As $x \rightarrow$ _____, $f(x) \rightarrow$ _____

Increasing Interval(s): _____

Decreasing Interval(s): _____

9. $f(x) = \sqrt[3]{x-1}$



D: _____ R: _____

Point of Inflection: _____

End Behavior:

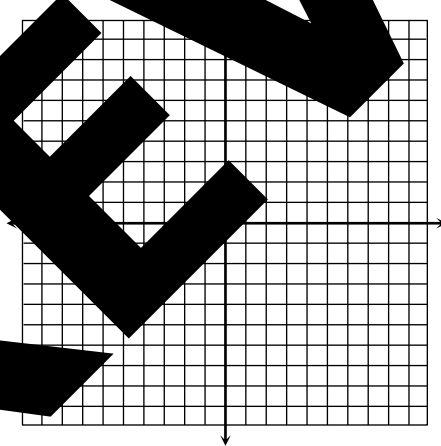
As $x \rightarrow$ _____, $f(x) \rightarrow$ _____

As $x \rightarrow$ _____, $f(x) \rightarrow$ _____

Increasing Interval(s): _____

Decreasing Interval(s): _____

10. $f(x) = -\sqrt[3]{x+4} - 2$



D: _____ R: _____

Point of Inflection: _____

End Behavior:

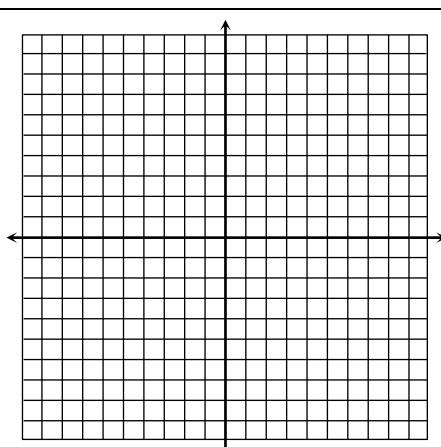
As $x \rightarrow$ _____, $f(x) \rightarrow$ _____

As $x \rightarrow$ _____, $f(x) \rightarrow$ _____

Increasing Interval(s): _____

Decreasing Interval(s): _____

11. $f(x) = \sqrt[3]{x} + 5$



D: _____ R: _____

Point of Inflection: _____

End Behavior:

As $x \rightarrow$ _____, $f(x) \rightarrow$ _____

As $x \rightarrow$ _____, $f(x) \rightarrow$ _____

Increasing Interval(s): _____

Decreasing Interval(s): _____

Name: _____

Algebra 2

Date: _____ Per: _____

Unit 6: Radical Functions

Quiz 6-3: Radical Equations & Graphing Radical Functions

Part I: Radical Equations – Solve each equation. Be sure to check for extraneous solutions.

1. $-2\sqrt{6x+3} = -18$

2. $\sqrt{m-7} + 18 = 12$

3. $(41k - 31)^{\frac{1}{4}} = 5$

4. $(45 - 2a)^{\frac{1}{3}} = 1$

5. $\sqrt[3]{19y + 40} = 3\sqrt[3]{y}$

6. $\sqrt{7n} = n$

7. $(21)^{\frac{1}{2}} = c - 7$

8. $p = \sqrt{4p + 8} - 3$

1. _____

3. _____

5. _____

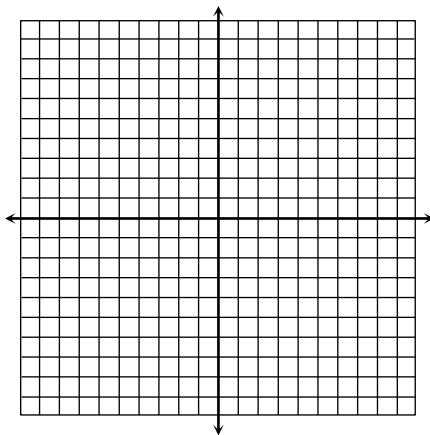
6. _____

7. _____

8. _____

Part II: Graphing – Graph each function and identify its key characteristics.

9. $f(x) = \sqrt{x} - 3$



D: _____ R: _____

Endpoint: _____

End Behavior:

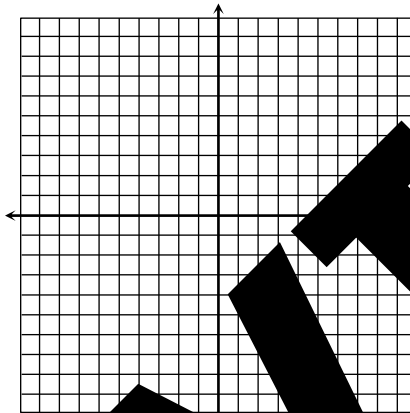
As $x \rightarrow$ _____, $f(x) \rightarrow$ _____

As $x \rightarrow$ _____, $f(x) \rightarrow$ _____

Increasing Interval(s): _____

Decreasing Interval(s): _____

10. $f(x) = -\sqrt{x+4} + 2$



D: _____ R: _____

Endpoint: _____

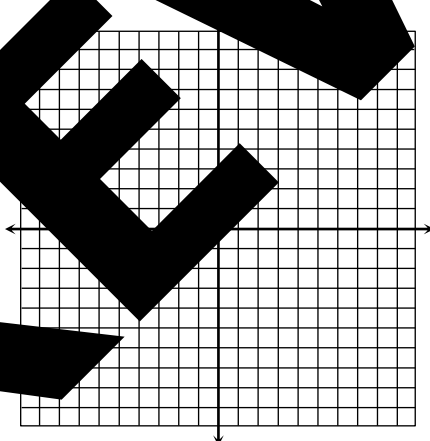
As $x \rightarrow$ _____, $f(x) \rightarrow$ _____

As $x \rightarrow$ _____, $f(x) \rightarrow$ _____

Increasing Interval(s): _____

Decreasing Interval(s): _____

11. $f(x) = 2\sqrt[3]{x-3} - 1$



D: _____ R: _____

Point of Inflection: _____

End Behavior:

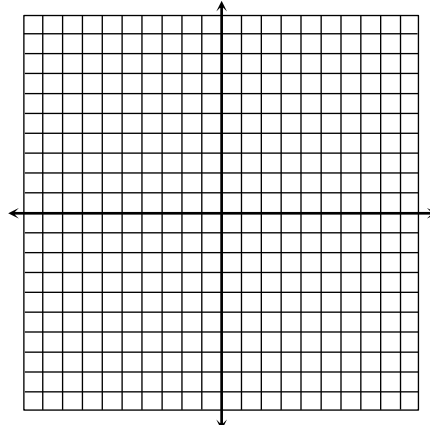
As $x \rightarrow$ _____, $f(x) \rightarrow$ _____

As $x \rightarrow$ _____, $f(x) \rightarrow$ _____

Increasing Interval(s): _____

Decreasing Interval(s): _____

12. $f(x) = \frac{3}{4}\sqrt[3]{x+1}$



D: _____ R: _____

Point of Inflection: _____

End Behavior:

As $x \rightarrow$ _____, $f(x) \rightarrow$ _____

As $x \rightarrow$ _____, $f(x) \rightarrow$ _____

Increasing Interval(s): _____

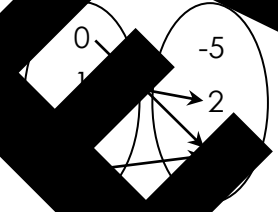
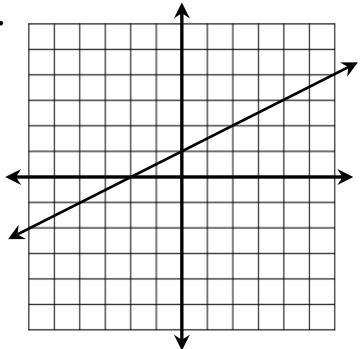
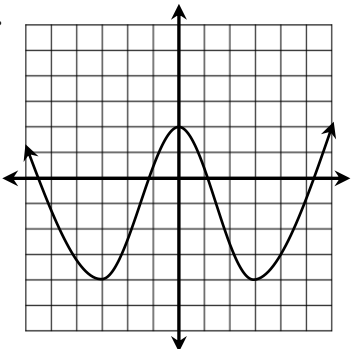
Decreasing Interval(s): _____

Name: _____

Date: _____

Topic: _____

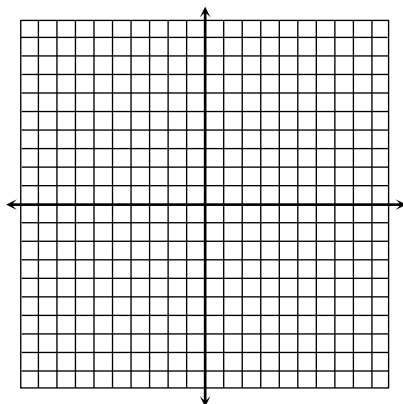
Class: _____

Main Ideas/Questions	Notes/Examples
INVERSE RELATIONS	Notation: The inverse of a relation R is written as _____. To find the inverse of a relation: _____
ONE-TO-ONE <i>Relationship</i>	
EXAMPLES	1. $R: \{(1, -2), (2, 4), (3, -1), (4, -2)\}$ R^{-1}: _____ Is R a function? _____ Is R^{-1} a function? _____ Is R one-to-one? _____ 2. $R: \{(1, 5), (1, 6), (3, 8), (4, 9)\}$ R^{-1}: _____ Is R a function? _____ Is R^{-1} a function? _____ Is R one-to-one? _____ 3. R:  Is R a function? _____ Is R^{-1} a function? _____ Is R one-to-one? _____
HORIZONTAL LINE TEST	Recall: The vertical line test is used to determine whether a relation is a function when given a graph. - The horizontal line test is used to determine whether the _____ of a relation is a function. BOTH the relation and its inverse must be a function in order to be a one-to-one function.
EXAMPLES Determine whether the relation shown represents a one-to-one function.	4.  5. 

INVERSE FUNCTIONS	Notation: The inverse of a function $f(x)$ is written as _____. To find the inverse of a function: » Step 1: Replace $f(x)$ with y. » Step 2: Interchange x with y. » Step 3: Solve for y. » Step 4: Rewrite y as $f^{-1}(x)$	
EXAMPLES Find the inverse of each function.	6. $f(x) = 5x$	7. $f(x) = x - 7$
	8. $f(x) = \frac{x-8}{3}$	9. $f(x) = \frac{2}{5}x + 1$
	10. $f(x) = x^2 + 8$ (if $x \geq 0$)	11. $f(x) = (x + 4)^3 - 6$
VERIFYING INVERSE FUNCTIONS using Composition	Two functions, f and g, are inverse functions if and only if: _____ AND _____	
EXAMPLES Determine whether the functions are inverse functions.	12. $f(x) = x^2 + 8$ (if $x \geq 0$) and $g(x) = \sqrt{x - 8}$	
	13. $f(x) = -4x + 5$ and $g(x) = \frac{x}{4} - 5$	

Graphing INVERSE FUNCTIONS

Given: $\triangle ABC$ is formed by the points in the relation $\{(-1, 6), (-7, 2), (-5, -3)\}$



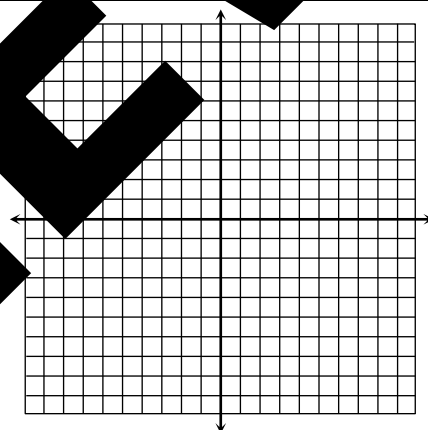
a) What is the inverse of this relation?

b) Graph $\triangle ABC$ and its inverse. What do you notice about the graph?

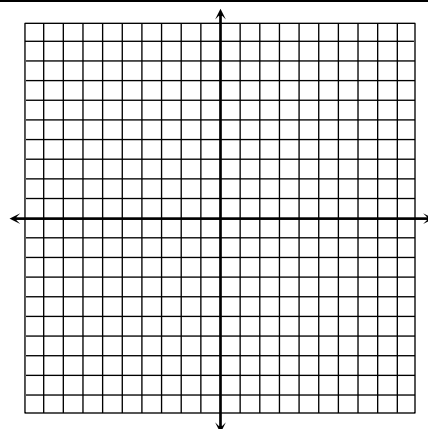
MORE EXAMPLES

Find the inverse of each function. Then, graph both functions and verify the inverse relationship on the graph.

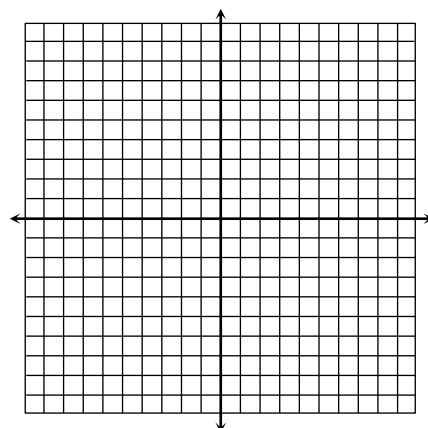
14. $f(x) = 2x + 8$



15. $f(x) = \frac{1}{3}x + 1$



16. $f(x) = x^2 - 4$ (if $x \geq 0$)



Name: _____

Unit 6: Radical Functions

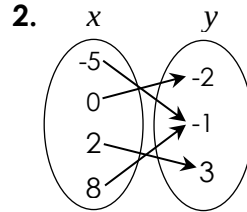
Date: _____ Per: _____

Homework 8: Inverse Relations & Functions

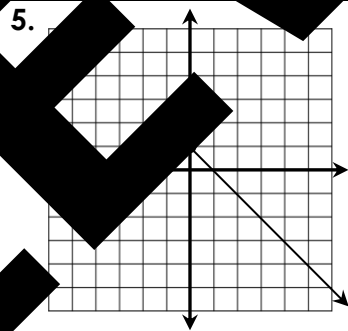
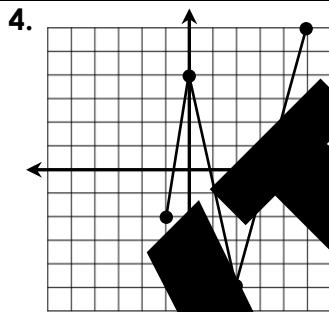
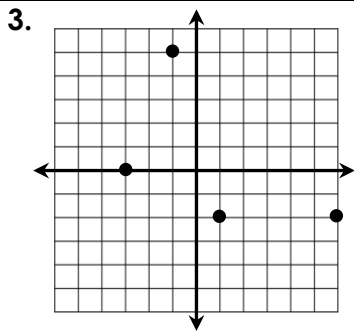
**** This is a 2-page document! ****

Give the inverse of each relation, then determine whether it is a one-to-one function.

1. $R = \{(-2, -7), (-1, -3), (0, 5), (1, 2)\}$



Determine whether the relation shown on the graph represents a one-to-one function.



Find the inverse of each function.

6. $f(x) = -x + 9$

$g(x) = \frac{4}{3}x - 7$

8. $f(x) = \sqrt{x}$

9. $f(x) = (x - 2)^4$ (if $x \geq 2$)

Determine whether the pair of functions are inverse functions.

10. $f(x) = x - 15$ and $g(x) = \frac{1}{6}x + \frac{5}{2}$

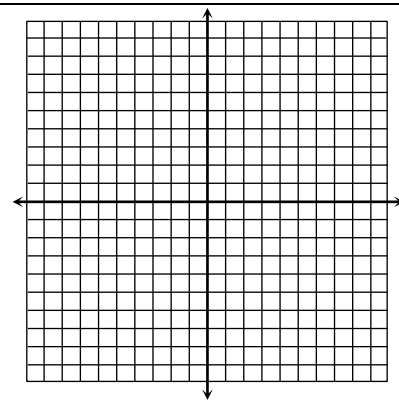
11. $f(x) = \frac{1}{2}x - \frac{7}{2}$ and $g(x) = 2x + 14$

12. $f(x) = \sqrt{x+4} + 9$ and $g(x) = (x-9)^2 - 4$ (if $x \geq 9$)

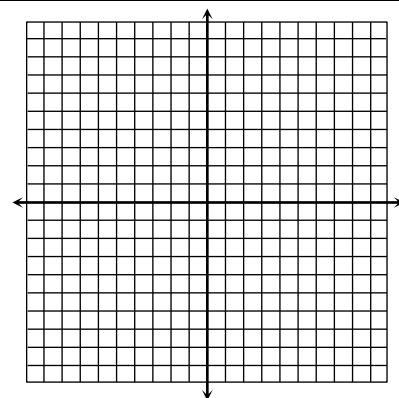
13. $f(x) = \frac{1}{4}x^3 - 3$ and $g(x) = \sqrt[3]{4x-12}$

Find the inverse of each function. Graph both functions and verify the inverse relationship on the graph.

14. $f(x) = -\frac{2}{3}x - 4$



$f(x) = x^2 + 6$ (if $x \geq 0$)



Unit 6 Test Study Guide (Radical Functions)

Name: _____

Date: _____ Per: _____

Topic 1: Simplifying Radicals

Perfect Squares: _____		
Perfect Cubes: _____		
Perfect Fourths: _____		
1. $-2\sqrt{294m^{16}n^7}$	2. $5\sqrt[3]{-80a^5}$	3. $-3\sqrt[4]{250}$

Topic 2: Operations with Radicals

Simplify.		
4. $-\sqrt{6} - 3\sqrt{45} + 2\sqrt{96}$	5. $\sqrt[3]{24} - 2\sqrt[4]{11} + \sqrt[4]{7}$	6. $\sqrt[3]{-12x^4y} \cdot \sqrt[3]{4x^2y^2}$
7. $\sqrt{18}(5 - \sqrt{2})$	8. $(\sqrt{6})(2\sqrt{3} - 5\sqrt{6})$	9. $(\sqrt{5} - 2)^2$
10.	11. $\frac{\sqrt{112a^6b^{12}}}{\sqrt{7a^2b^3}}$	12. $\frac{\sqrt[3]{7}}{\sqrt[3]{56}}$

13. $\frac{5\sqrt{6}}{\sqrt{15}}$	14. $\sqrt{\frac{10m^3}{18m}}$	15. $\frac{\sqrt{5}-\sqrt{2}}{4\sqrt{2}}$
16. $\frac{2}{5+3\sqrt{2}}$	17. $\frac{2+\sqrt{7}}{5-\sqrt{7}}$	

Topic 3: Rational Exponents

Rewrite in radical form. Simplify if possible.			
18. $16^{\frac{1}{4}}$	19. $a^{\frac{2}{3}}$	20. $(2y)^2$	21. $18m^{\frac{7}{4}}$
Rewrite in exponential form.			
22. $\sqrt[3]{17m^2}$	23. $\sqrt[5]{27}$	24. $\sqrt{12a^9b^8}$	25. $\sqrt[6]{4r^2s^{10}}$
Simplify each expression. Final answers in simplest radical form.			
	27. $(81^8)^{\frac{1}{4}}$	28. $\frac{k^{\frac{13}{3}}}{k}$	
29. $\sqrt{m^9} \cdot m$	30. $\frac{2^3}{\sqrt{2^7}}$	31. $\frac{\sqrt{x^3} \cdot \sqrt[4]{x^3}}{x}$	

Topic 4: Solving Radical Equations

Solve each equation. Check for extraneous solutions.

32. $\sqrt{5x-1} + 4 = 11$

33. $1 - 2\sqrt{m+12} = 5$

34. $(9a + 45)^{\frac{1}{3}} = 3$

35. $\sqrt{3n-2} + \sqrt{43-2n} = 10$

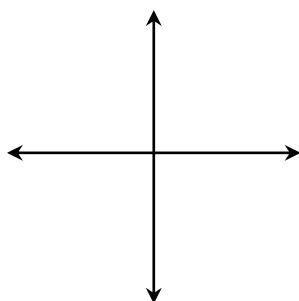
36. $(18+7k)^{\frac{1}{2}} = k$

$\sqrt{x+19} - 4 = x$

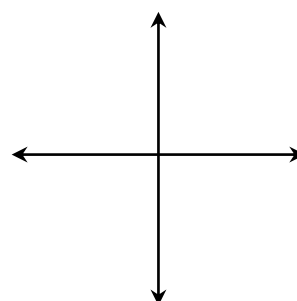
Topic 5: Graphing Radical Functions

Identify the parent function and sketch the shape of the graph.

SQUARE ROOT



CUBE ROOT



Describe the transformations on each function compared to its parent function.

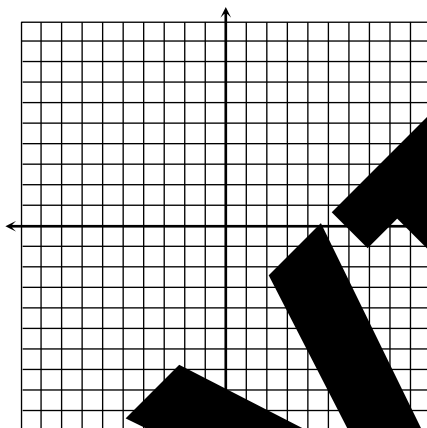
38. $f(x) = \frac{3}{4}\sqrt{x} + 1$

39. $f(x) = 2\sqrt[3]{x+5} - 8$

40. The cube root parent function is reflected about the x-axis, then shifted so that its point of inflection is located at $(-6, -2)$. Write an equation that represents this new function.

Graph each function and identify its key characteristics.

41. $f(x) = 4\sqrt{x} - 7$



D: _____ R: _____

Endpoint: _____

End Behavior: _____

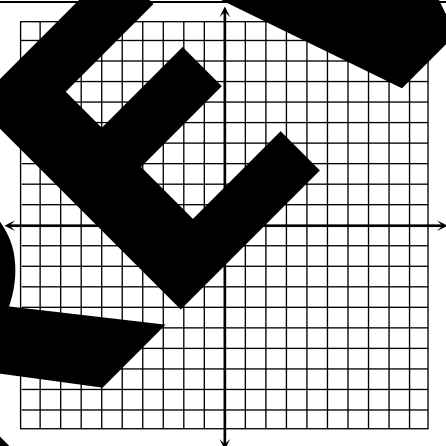
As $x \rightarrow$ _____, $f(x) \rightarrow$ _____

As $x \rightarrow$ _____, $f(x) \rightarrow$ _____

Increasing Interval(s): _____

Decreasing Interval(s): _____

42. $f(x) = -\sqrt{x+6} + 5$



D: _____ R: _____

Endpoint: _____

End Behavior: _____

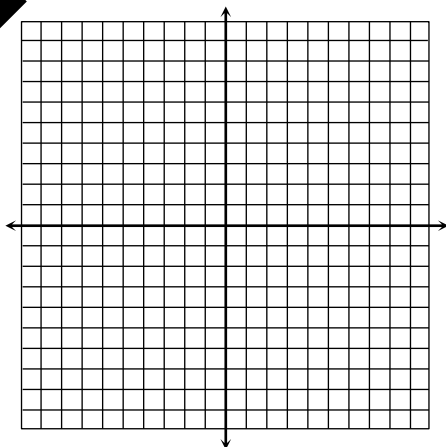
As $x \rightarrow$ _____, $f(x) \rightarrow$ _____

As $x \rightarrow$ _____, $f(x) \rightarrow$ _____

Increasing Interval(s): _____

Decreasing Interval(s): _____

43. $f(x) = \sqrt{x-1}$



D: _____ R: _____

Point of Inflection: _____

End Behavior: _____

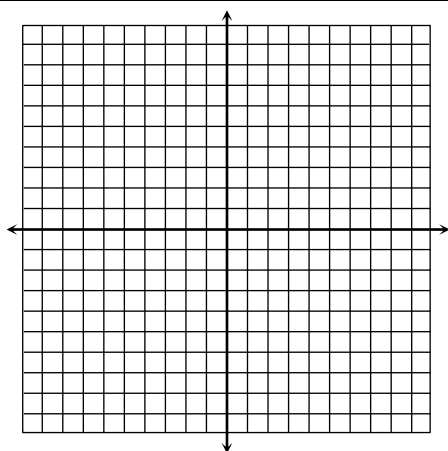
As $x \rightarrow$ _____, $f(x) \rightarrow$ _____

As $x \rightarrow$ _____, $f(x) \rightarrow$ _____

Increasing Interval(s): _____

Decreasing Interval(s): _____

44. $f(x) = \frac{1}{2}\sqrt[3]{x+3} - 5$



D: _____ R: _____

Point of Inflection: _____

End Behavior:

As $x \rightarrow$ _____, $f(x) \rightarrow$ _____

As $x \rightarrow$ _____, $f(x) \rightarrow$ _____

Increasing Interval(s): _____

Decreasing Interval(s): _____

Topic 6: Inverse Functions

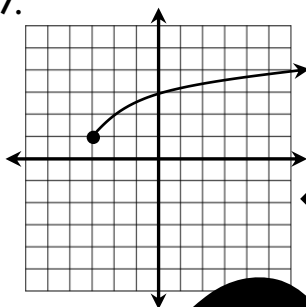
Find the inverse of each relation. Determine if it is a one-to-one function.

45. $R = \{(5, 1), (-2, -3), (5, 9), (2, 7)\}$

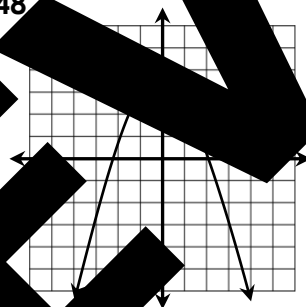
46. $R = \{(-4, -3), (-4, 4), (-5, 5)\}$

Determine whether the relations below represent one-to-one functions.

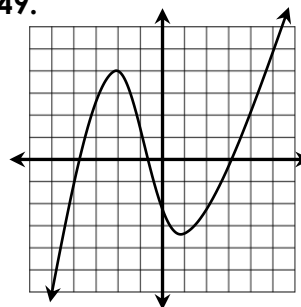
47.



48.

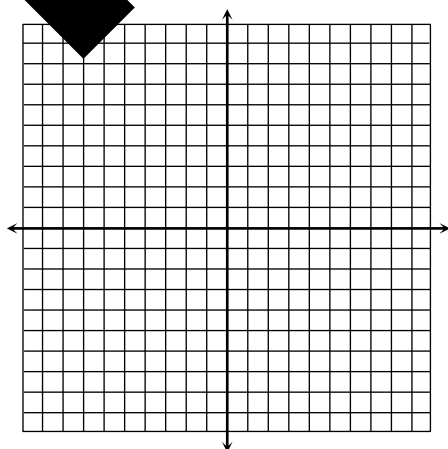


49.

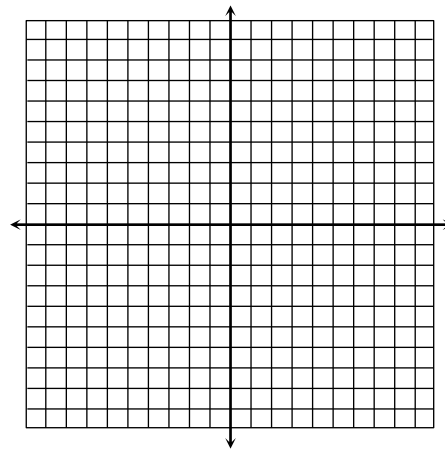


Find the inverse of each function. Verify the inverse relationship on the graph.

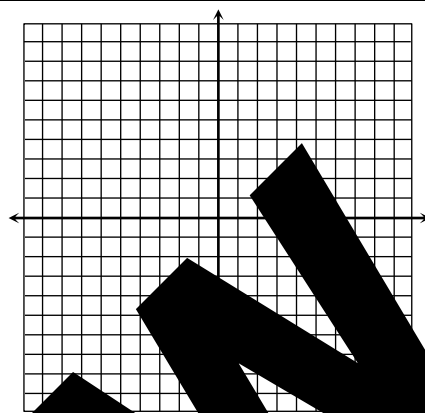
50. $f(x) =$



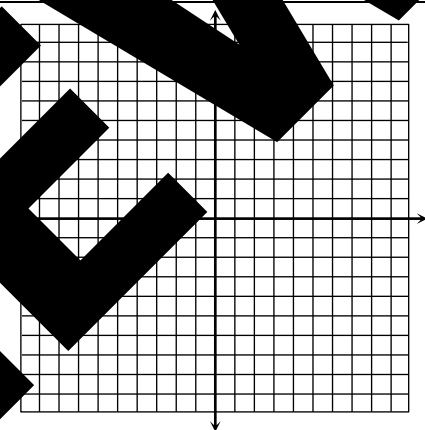
51. $f(x) = x + 7$



52. $f(x) = \frac{2}{5}x - 1$



53. $f(x) = \sqrt{x} + 3$



Determine whether the pair of functions are inverse functions.

54. $f(x) = 8x - 12$ and $g(x) = \frac{1}{8}x + \frac{3}{2}$

55. $f(x) = \frac{1}{9}x^3 - 2$ and $g(x) = \sqrt[3]{2x + 4}$

56. $f(x) = \frac{1}{9}x^3 - 2$ and $g(x) = \sqrt[3]{2x + 4}$

Name: _____

Date: _____ Per: _____

Unit 6 Test

Radical Functions

1. Simplify the expression below.

$$\sqrt[3]{81m^8n^4}$$

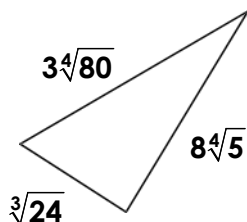
- A. $9m^2n\sqrt[3]{n}$
- B. $9m^3n\sqrt[3]{m^2n}$
- C. $3m^2n\sqrt[3]{3n}$
- D. $3m^2n\sqrt[3]{3m^2n}$

2. Simplify the expression below.

$$5xy\sqrt{28x} + \sqrt{63x^3y^2}$$

- A. $13xy\sqrt{7x}$
- B. $13x^2y^2$
- C. $29xy$
- D. $29x^2y^2\sqrt{7x}$

3. Find the perimeter of the triangle below.
Write your answer in simplest radical form.



4. Simplify the expression below.

$$\sqrt[4]{8w^3} \cdot 3\sqrt[4]{32w^5}$$

- A. $24w^2$
- B. $96w^2$
- C. $24w^3\sqrt[4]{w^3}$
- D. $96w^3\sqrt[4]{w^3}$

5. Simplify the expression below.

$$\sqrt{18}(\sqrt{3}-5) - 2\sqrt{3}$$

6. Simplify the expression below.

$$(4\sqrt{3}-1)(\sqrt{3}-6)$$

7. Simplify the expression below.

$$(2\sqrt{10} - \sqrt{6})(2\sqrt{10} + \sqrt{6})$$

8. The side length of a square is $(\sqrt{2} - 3\sqrt{5})$ meters. Find the area of the square and write your answer in simplest radical form.

9. Simplify the expression below.

$$\frac{\sqrt{294k^{30}}}{\sqrt{3k^5}}$$

- A. $7k^2\sqrt{2}$
- B. $7k^3\sqrt{2}$
- C. $7k^5\sqrt{2}$
- D. $7k^{12}\sqrt{2k}$

10. Simplify the expression below.

- A. $-\frac{10}{10}$
- B. $-\frac{10}{10}$
- C. $-\frac{10}{10}$
- D. $-\frac{1}{10}$

11. Simplify the expression below.

$$\frac{12\sqrt{3}}{\sqrt{8}}$$

12. Simplify the expression below.

$$\sqrt{\frac{5x}{12}}$$

- A. $\frac{\sqrt{5x}}{6}$
- B. $\frac{\sqrt{15x}}{6}$
- C. $4\sqrt{15x}$
- D. $\frac{\sqrt{15x}}{3}$

13. Simplify the expression below.

$$\frac{4\sqrt{5} + 4}{\sqrt{2}}$$

14. Simplify the expression below.

$$\frac{3 + 5\sqrt{7}}{3 - \sqrt{7}}$$

15. Which is equivalent to the expression below?

$$\sqrt{16x^7y^4}$$

A. $4x^{\frac{2}{7}}y^{\frac{1}{2}}$

C. $8x^{\frac{2}{7}}y^{\frac{1}{2}}$

B. $4x^{\frac{7}{2}}y^2$

D. $8x^{\frac{7}{2}}y^2$

16. Which is equivalent to the expression below?

$$a^{\frac{5}{4}}b^{\frac{13}{4}}$$

A. $\frac{1}{4}\sqrt{a^5b^{13}}$

C. $a^2b^{\frac{13}{4}}\sqrt{ab}$

B. $a^4b^4\sqrt{ab}$

D. $ab^{\frac{13}{4}}$

17. Simplify the expression below.

$$(2x)^{\frac{1}{3}} \cdot (2x)^{\frac{4}{3}}$$

A. $2x\sqrt[3]{4x^2}$

B. $2x\sqrt[3]{2x}$

C. $4x\sqrt[3]{4x^2}$

D. $4x\sqrt[3]{2x}$

18. Simplify the expression below.

$$\frac{9^{\frac{7}{4}}}{9^{\frac{9}{4}}}$$

19. Simplify the expression below.

$$\left(\frac{3}{p^2}\right)^{\frac{7}{2}}$$

A. p^5

B. $p^2\sqrt{p}$

C. $p^{\frac{5}{2}}$

D. $p^{10}\sqrt{p}$

Simplify the expression below.

$$\frac{(-54)^2}{\sqrt[3]{(-54)^5}}$$

Solve the equation below.

$$\sqrt{6n-5}+1=8$$

22. Solve the equation below.

$$-5\sqrt[3]{2x+3}=-15$$

23. Solve the equation below.

$$7 - (3k - 29)^{\frac{1}{4}} = 5$$

24. Solve the equation below.

$$(7 - 2c)^{\frac{1}{2}} = (1 - 5c)^{\frac{1}{2}}$$

25. Solve the equation below.

$$n = \sqrt{13n - 40}$$

26. Solve the equation below.

$$\sqrt{y + 15} = y$$

27. The square root parent function is translated so that the endpoint is located at $(-7, 1)$. Write an equation that represents this function.

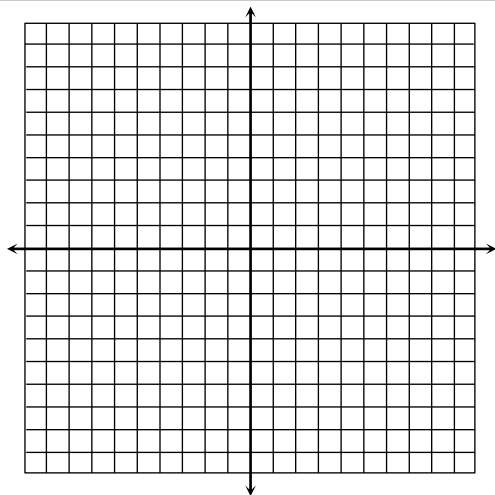
28. The transformations that take place by graphing the function below in respect to its parent function are. Check all that apply.

$$f(x) = -\frac{2}{3}\sqrt[3]{x + 9} - 4$$

☐ Up 9 units	☐ Up 4 units	☐ Vertical Stretch
☐ Down 9 units	☐ Down 4 units	☐ Vertical Compression
☐ Left 9 units	☐ Left 4 units	☐ Reflection in x-axis
☐ Right 9 units	☐ Right 4 units	☐ Reflection in y-axis

Graph each function and identify its key characteristics.

29. $f(x) = \sqrt{x-7} + 2$



D: _____ R: _____

Endpoint: _____

End Behavior:

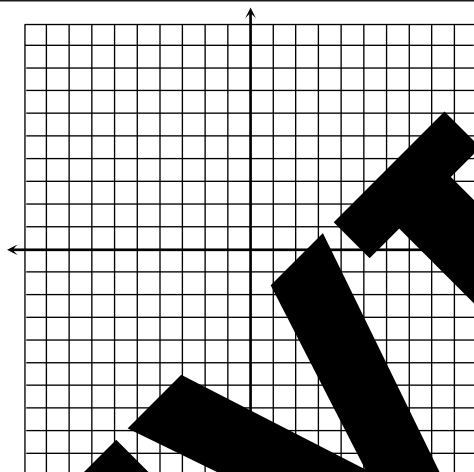
As $x \rightarrow$ _____, $f(x) \rightarrow$ _____

As $x \rightarrow$ _____, $f(x) \rightarrow$ _____

Increasing Interval(s): _____

Decreasing Interval(s): _____

30. $f(x) = 2\sqrt[3]{x} - 4$



D: _____ R: _____

Endpoint: _____

End Behavior:

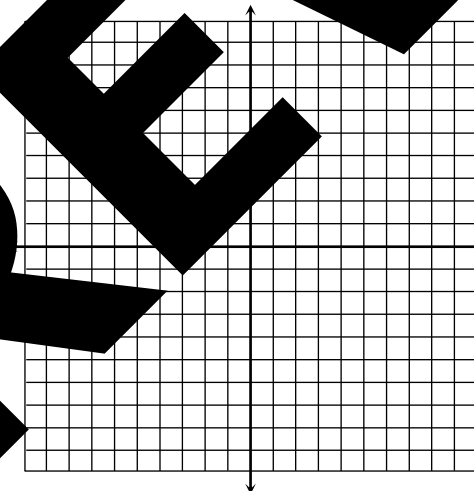
As $x \rightarrow$ _____, $f(x) \rightarrow$ _____

As $x \rightarrow$ _____, $f(x) \rightarrow$ _____

Increasing Interval(s): _____

Decreasing Interval(s): _____

31. $f(x) = -\frac{1}{2}\sqrt{x} - 3$



D: _____ R: _____

Endpoint: _____

End Behavior:

As $x \rightarrow$ _____, $f(x) \rightarrow$ _____

As $x \rightarrow$ _____, $f(x) \rightarrow$ _____

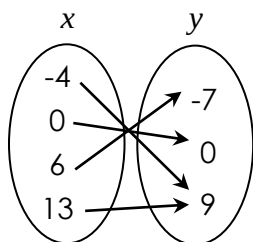
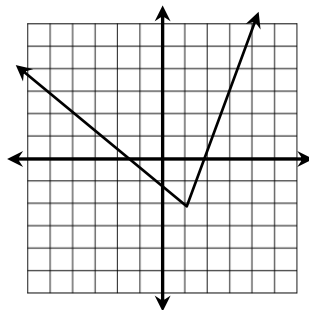
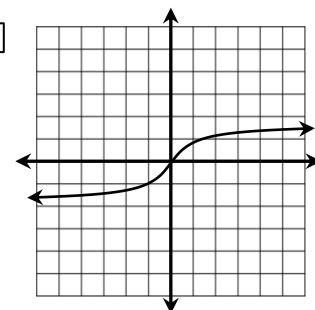
Increasing Interval(s): _____

Decreasing Interval(s): _____

32. Which of the following relations represent a one-to-one function? Check all that apply.

☐

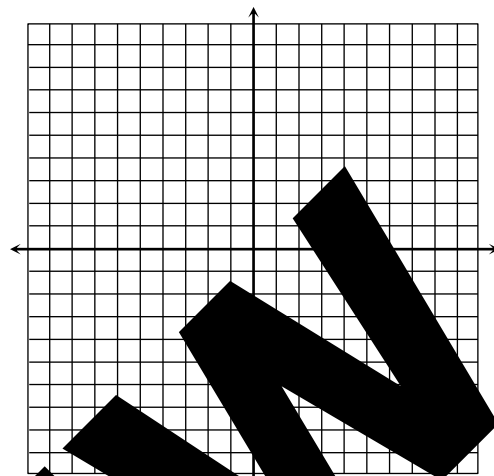
x	y
-2	-10
-1	-5
1	5
2	10

☐

☐

☐


33. Write the inverse of the function below. Graph both functions to verify the relationship.

$$f(x) = -\frac{1}{4}x - 2$$

Inverse Function:



Determine whether the functions below are inverse functions. Provide your answer below.

34. $f(x) = \frac{1}{3}x - 3$ and $g(x) = 6x + 18$

☐ Yes

☐ No

35. $f(x) = \sqrt[4]{2x - 6}$ and $g(x) = \frac{1}{2}x^4 + 3$

☐ Yes

☐ No

BONUS: Solve for x: $-1 + 1 = 2x$

Name:		Date:	
Topic:		Class:	

Main Ideas/Questions	Notes/Examples																				
WARM-UP List the perfect squares, cubes, and fourths.	Perfect Squares: $1, 4, 9, 16, 25, 36, 49, 64, 81, 100, \dots$ Perfect Cubes: $1, 8, 27, 64, 125, 216, 343, 512, 729, 1000, \dots$ Perfect Fourths: $1, 16, 81, 256, 625, 1296, 2401, 4096, 6561, 10000, \dots$																				
NTH ROOTS	Definition: x is the n^{th} root of a real number a if $x^n = a$ Examples: $\underline{9}$ and $\underline{-9}$ are square roots of 81 because $9^2 = 81$ and $(-9)^2 = 81$ $\underline{-2}$ is the cube root of -8 because $(-2)^3 = -8$ $\underline{4}$ and $\underline{-4}$ are fourth roots of 256 because $4^4 = 256$ and $(-4)^4 = 256$																				
RADICAL Expression	The n^{th} root of a real number, a, can be written as the radical expression $\sqrt[n]{a}$ If there is no index, it is understood that $n = 2$. <table border="1" style="width: 100%; border-collapse: collapse; margin-top: 10px;"> <thead> <tr> <th>Index</th> <th>Radicand</th> <th>Type of Roots</th> <th># of Roots</th> </tr> </thead> <tbody> <tr> <td>Even</td> <td>Positive</td> <td>real</td> <td>$2(\pm)$</td> </tr> <tr> <td>Odd</td> <td>Positive</td> <td>real</td> <td>$1(+)$</td> </tr> <tr> <td>Odd</td> <td>Negative</td> <td>real</td> <td>$1(-)$</td> </tr> <tr> <td>★ Even</td> <td>Negative</td> <td>imaginary</td> <td>$2(\pm)$</td> </tr> </tbody> </table> If a radicand has more than one n^{th} root, the radical sign indicates only the principal, or positive, root.	Index	Radicand	Type of Roots	# of Roots	Even	Positive	real	$2(\pm)$	Odd	Positive	real	$1(+)$	Odd	Negative	real	$1(-)$	★ Even	Negative	imaginary	$2(\pm)$
Index	Radicand	Type of Roots	# of Roots																		
Even	Positive	real	$2(\pm)$																		
Odd	Positive	real	$1(+)$																		
Odd	Negative	real	$1(-)$																		
★ Even	Negative	imaginary	$2(\pm)$																		
EVALUATING Radicals	Find each value. <table style="width: 100%; text-align: center;"> <tr> <td>$\sqrt{16} = 4$</td> <td>$-\sqrt{121} = -11$</td> <td>$\sqrt{289} = 17$</td> <td>$-\sqrt{\frac{4}{25}} = -\frac{2}{5}$</td> </tr> <tr> <td>$\sqrt[3]{8} = 2$</td> <td>$\sqrt[3]{343} = 7$</td> <td>$\sqrt[3]{-125} = -5$</td> <td>$\sqrt[3]{-\frac{1}{27}} = -\frac{1}{3}$</td> </tr> <tr> <td>$-\sqrt[4]{1} = -1$</td> <td>$\sqrt{2,401} = 49$</td> <td>$-\sqrt[4]{4,096} = -8$</td> <td>$\sqrt[4]{\frac{81}{16}} = \frac{3}{2}$</td> </tr> </table>	$\sqrt{16} = 4$	$-\sqrt{121} = -11$	$\sqrt{289} = 17$	$-\sqrt{\frac{4}{25}} = -\frac{2}{5}$	$\sqrt[3]{8} = 2$	$\sqrt[3]{343} = 7$	$\sqrt[3]{-125} = -5$	$\sqrt[3]{-\frac{1}{27}} = -\frac{1}{3}$	$-\sqrt[4]{1} = -1$	$\sqrt{2,401} = 49$	$-\sqrt[4]{4,096} = -8$	$\sqrt[4]{\frac{81}{16}} = \frac{3}{2}$								
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SIMPLIFYING

Radicals

$$1. \sqrt{117} = \sqrt{9 \cdot 13}$$

$$= 3\sqrt{13}$$

$$2. 4\sqrt{320} = 4 \cdot \sqrt{64} \cdot \sqrt{5}$$

$$= 4 \cdot 8 \cdot \sqrt{5}$$

$$= 32\sqrt{5}$$

$$3. 2\sqrt[3]{48} = 2 \cdot \sqrt[3]{8} \cdot \sqrt[3]{6}$$

$$= 2 \cdot 2 \cdot \sqrt[3]{6}$$

$$= 4\sqrt[3]{6}$$

$$4. 3\sqrt[3]{108} = 3 \cdot \sqrt[3]{27} \cdot \sqrt[3]{4}$$

$$= 3 \cdot 3 \cdot \sqrt[3]{4}$$

$$= 9\sqrt[3]{4}$$

$$5. \sqrt[3]{-250} = \sqrt[3]{-125} \cdot \sqrt[3]{2}$$

$$= -5\sqrt[3]{2}$$

$$6. \sqrt[3]{-54} = \sqrt[3]{-27} \cdot \sqrt[3]{2}$$

$$= -3 \cdot \sqrt[3]{2}$$

$$= -3\sqrt[3]{2}$$

$$7. 3\sqrt[4]{162} = 3 \cdot \sqrt[4]{81} \cdot \sqrt[4]{2}$$

$$= 3 \cdot 3 \cdot \sqrt[4]{2}$$

$$= 9\sqrt[4]{2}$$

$$8. 5\sqrt[4]{1296} = 5 \cdot \sqrt[4]{1296} \cdot \sqrt[4]{2}$$

$$= 5 \cdot 6 \cdot \sqrt[4]{2}$$

$$= 30\sqrt[4]{2}$$

Radicals with VARIABLES

Square Roots
Exponents must be multiples of 2

Cube Roots
Exponents must be multiples of 3

4th Roots
Exponents must be multiples of 4

$$9. \sqrt{16x^4y^8} = \sqrt{16x^4y^8} \cdot \sqrt{2y}$$

$$= 4x^2y^4\sqrt{2y}$$

$$10. \sqrt{324a^3b^7} = \sqrt{324a^2b^6} \cdot \sqrt{ab}$$

$$= 18ab^3\sqrt{ab}$$

$$11. \sqrt[3]{216m^3n^6} = 6mn^2$$

$$12. \sqrt[3]{56r^6s^4} = \sqrt[3]{8r^6s^3} \cdot \sqrt[3]{7r^2s}$$

$$= 2r^2s\sqrt[3]{7r^2s}$$

$$13. \sqrt[3]{-64x^9y^{21}} = \sqrt[3]{-64x^9y^{21}} \cdot \sqrt[3]{x}$$

$$= -4x^3y^7\sqrt[3]{x}$$

$$14. \sqrt[3]{-81p^2q^{12}} = \sqrt[3]{-27q^{12}} \cdot \sqrt[3]{3p^2}$$

$$= -3q^4\sqrt[3]{3p^2}$$

$$15. \sqrt[4]{w^4v^{16}} = \sqrt[4]{w^4v^{16}} \cdot \sqrt[4]{v}$$

$$= wv^4\sqrt[4]{v}$$

$$16. \sqrt[4]{48m^8n^3} = \sqrt[4]{16m^8} \cdot \sqrt[4]{3n^3}$$

$$= 2m^2\sqrt[4]{3n^3}$$

Name: _____

Unit 6: Radical Functions

Date: _____ Per: _____

Homework 1: n^{th} Roots; Simplifying Radicals

For each value, give the number of possible roots, then list the roots.

1. 196, square root(s)

2 real roots;

$$\sqrt{196} = \pm 14$$

2. 216, cube root(s)

1 real root;

$$\sqrt[3]{216} = 6$$

3. 625, fourth root(s)

2 real roots;

$$\sqrt[4]{625} = \pm 5$$

Evaluate each expression

4. $\sqrt{225} = 15$

5. $\sqrt[3]{512} = 8$

6. $\sqrt[3]{-64} = -4$

Simplify each radical expression.

8. $5\sqrt{150} = 5 \cdot \sqrt{25} \cdot \sqrt{6}$

$$= 5 \cdot 5 \cdot \sqrt{6}$$

$$= 25\sqrt{6}$$

9. $-8\sqrt{63} = -8\sqrt{9} \sqrt{7}$

$$= -8 \cdot 3 \cdot \sqrt{7}$$

$$= -24\sqrt{7}$$

10. $\sqrt[3]{125} = \sqrt[3]{125} \cdot \sqrt[3]{1}$

$$= 5\sqrt[3]{1}$$

$$= 5$$

11. $2\sqrt[3]{-54} = 2 \cdot \sqrt[3]{-27} \cdot \sqrt[3]{2}$

$$= 2 \cdot (-3) \cdot \sqrt[3]{2}$$

$$= -6\sqrt[3]{2}$$

12. $-3\sqrt[4]{567} = -3 \cdot \sqrt[4]{81} \cdot \sqrt[4]{7}$

$$= -3 \cdot 3 \cdot \sqrt[4]{7}$$

$$= -9\sqrt[4]{7}$$

13. $6\sqrt[4]{405} = 6 \cdot \sqrt[4]{81} \cdot \sqrt[4]{5}$

$$= 6 \cdot 3 \cdot \sqrt[4]{5}$$

$$= 18\sqrt[4]{5}$$

14. $3\sqrt{252k^5} = 3 \cdot \sqrt{36k^4} \cdot \sqrt{7k}$

$$= 3 \cdot 6k^2 \cdot \sqrt{7k}$$

$$= 18k^2\sqrt{7k}$$

15. $4\sqrt{60a^4b} = 4 \cdot \sqrt{15} \cdot \sqrt{4a^3b}$

$$= 4 \cdot \sqrt{15} \cdot 2a^3 \sqrt{b}$$

$$= 8a^3\sqrt{15b}$$

16. $3\sqrt{49x^9y^{16}} = 3 \cdot \sqrt{49x^8y^{16}} \cdot \sqrt{x}$

$$= 3 \cdot 7x^4y^8 \sqrt{x}$$

$$= 21x^4y^8\sqrt{x}$$

$$= 21x^4y^8\sqrt{x}$$

17. $\sqrt[3]{192p^7q^6} = \sqrt[3]{64p^6q^6} \cdot \sqrt[3]{3p}$

$$= 4p^2q^2 \sqrt[3]{3p}$$

$$= 4p^2q^2 \sqrt[3]{3p}$$

$$= 8p^2q^2 \sqrt[3]{3p}$$

18. $\sqrt[3]{-r^{14}s^9} = \sqrt[3]{-r^{12}s^9} \cdot \sqrt[3]{r^2}$

$$= -r^4s^3 \sqrt[3]{r^2}$$

$$= -r^4s^3 \sqrt[3]{r^2}$$

$$= -r^4s^3 \sqrt[3]{r^2}$$

19. $\sqrt[3]{-625m^{28}n^{12}} = \sqrt[3]{-125m^{27}n^{12}} \cdot \sqrt[3]{5m}$

$$= -5m^9n^4 \sqrt[3]{5m}$$

$$= -5m^9n^4 \sqrt[3]{5m}$$

$$= -5m^9n^4 \sqrt[3]{5m}$$

20. $\sqrt[4]{81m^4n^{12}} = \sqrt[4]{81m^4n^{12}} \cdot \sqrt[4]{3mn}$

$$= 3mn^3 \sqrt[4]{3mn}$$

$$= 3mn^3 \sqrt[4]{3mn}$$

21. $-2\sqrt[4]{80x^8y^9z} = -2 \cdot \sqrt[4]{16x^8y^8} \cdot \sqrt[4]{5yz}$

$$= -2 \cdot 2x^2y^2 \cdot \sqrt[4]{5yz}$$

$$= -4x^2y^2 \sqrt[4]{5yz}$$

$$= -4x^2y^2 \sqrt[4]{5yz}$$

22. $5\sqrt[4]{112c^{24}d^{13}} = 5 \cdot \sqrt[4]{16c^{24}d^{12}} \cdot \sqrt[4]{7d}$

$$= 5 \cdot 2c^6d^3 \cdot \sqrt[4]{7d}$$

$$= 10c^6d^3 \sqrt[4]{7d}$$

$$= 10c^6d^3 \sqrt[4]{7d}$$

Name:	Date:
Topic:	Class:

Main Ideas/Questions	Notes/Examples		
RATIONAL EXPONENTS	Expressions with rational exponents can be rewritten as radicals using the following rule:		
	Exponential Form	Meaning	Radical Form
	$a^{\frac{1}{n}}$	The n^{th} root of a	$\sqrt[n]{a}$
Converting EXPONENTIAL TO RADICAL FORM	Directions: Write each expression in radical form. Simplify if needed.		
	1. $x^{\frac{1}{4}}$ $= \sqrt[4]{x}$	2. $24^{\frac{1}{3}}$ $= \sqrt[3]{24}$	3. $(15x)^{\frac{1}{2}}$ $= \sqrt{15x}$
	4. $7^{\frac{2}{3}}$ $= \sqrt[3]{7^2}$	5. $k^{\frac{7}{2}}$ $= \sqrt{k^7}$ $= \sqrt{k^6 \cdot k}$ $= k^3 \sqrt{k}$	6. $3^{\frac{5}{4}}$ $= \sqrt[4]{3^5}$ $= \sqrt[4]{3^4 \cdot 3}$ $= 3 \sqrt[4]{3}$
	7. $(ab)^{\frac{3}{4}}$ $= \sqrt[4]{(ab)^3}$	8. $(-6x)^{\frac{2}{3}}$ $= \sqrt[3]{(-6x)^2}$ $= \sqrt[3]{36x^2}$	9. $7(12w)^{\frac{1}{2}}$ $= 7 \sqrt{12w}$ $= 7 \sqrt{4 \cdot 3w}$ $= 14 \sqrt{3w}$
	Directions: Write each expression in exponential form.		
	10. $16^{\frac{1}{3}}$ $= 16^{\frac{1}{3}}$	11. $\sqrt{xy}$ $= (xy)^{\frac{1}{2}}$ $= x^{\frac{1}{2}} y^{\frac{1}{2}}$	12. $\sqrt[4]{8w^2}$ $= (8w^2)^{\frac{1}{4}}$ $= 8^{\frac{1}{4}} w^{\frac{1}{2}}$
Converting RADICAL TO EXPONENTIAL FORM	13. $\sqrt[3]{11^2}$ $= 11^{\frac{2}{3}}$	14. $\sqrt[4]{k^{10}}$ $= k^{\frac{10}{4}}$ $= k^{\frac{5}{2}}$	15. $(\sqrt{3m})^7$ $= (3m)^{\frac{7}{2}}$ $= 3^{\frac{7}{2}} m^{\frac{7}{2}}$
	16. $(\sqrt[4]{-2a})^5$ $= (-2a)^{\frac{5}{4}}$ $= (-2)^{\frac{5}{4}} \cdot a^{\frac{5}{4}}$	17. $\sqrt{10^5 a^3 b^8}$ $= (10^5 a^3 b^8)^{\frac{1}{2}}$ $= 10^{\frac{5}{2}} a^{\frac{3}{2}} b^4$	18. $\sqrt[8]{9x^2 y^{12}}$ $= (9x^2 y^{12})^{\frac{1}{8}}$ $= 9^{\frac{1}{8}} x^{\frac{1}{4}} y^{\frac{3}{2}}$

SIMPLIFYING EXPRESSIONS

with Rational
Exponents

- ① Rewrite all radicals in **exponential form**.
- ② Use the **exponent rules** to simplify the expression.
- ③ Write your **answer as a radical in simplest form**. Simplify if needed.

$$19. x^{\frac{1}{3}} \cdot x^{\frac{4}{3}} = x^{5/3}$$

$$= \sqrt[3]{x^5}$$

$$= \sqrt[3]{x^3 \cdot x^2} = \boxed{x \sqrt[3]{x^2}}$$

$$20. p^{\frac{1}{4}} \cdot p^{\frac{3}{4}} = p^{\frac{1}{4} + \frac{3}{4}} = p^1 = p$$

$$= \sqrt[4]{p^1} = \sqrt[4]{p}$$

$$21. \frac{m^{\frac{5}{2}}}{m^{\frac{7}{4}}} = \frac{m^{\frac{10}{4}}}{m^{\frac{7}{4}}} = m^{\frac{3}{4}}$$

$$= \sqrt[4]{m^3}$$

$$22. (a^5)^{\frac{1}{2}}$$

$$= a^{\frac{5}{2}} = \sqrt{a^5}$$

$$23. (32^{\frac{1}{2}})^{\frac{1}{2}} = 32^{\frac{1}{4}}$$

$$= \sqrt[4]{32}$$

$$= \sqrt[4]{16 \cdot 2} = \sqrt[4]{2^4 \cdot 2} = \boxed{2 \sqrt[4]{2}}$$

$$24. 8^{\frac{2}{3}} \cdot x^{\frac{4}{3}} = 8^{\frac{2}{3}} \cdot x^{\frac{4}{3}}$$

$$= \sqrt[3]{64} \cdot \sqrt[3]{x^4} = \boxed{4x \sqrt[3]{x}}$$

$$25. 100^{-\frac{1}{2}} = \frac{1}{100^{\frac{1}{2}}}$$

$$= \frac{1}{10}$$

$$26. 16^{\frac{1}{3}} \cdot 16^{\frac{2}{3}} = 16^{\frac{1}{3} + \frac{2}{3}} = 16^1 = 16$$

$$= \sqrt[3]{16}$$

$$= \sqrt[3]{8 \cdot 2} = \boxed{2 \sqrt[3]{2}}$$

$$27. \sqrt[3]{216} = \sqrt[3]{2^3 \cdot 3^3} = \sqrt[3]{2^3 \cdot 3^3} = \boxed{6}$$

$$28. \left(\frac{112}{7}\right)^{-\frac{1}{4}} = \left(\frac{16}{1}\right)^{-\frac{1}{4}} = \left(\frac{1}{16}\right)^{\frac{1}{4}} = \sqrt[4]{\frac{1}{16}} = \boxed{\frac{1}{2}}$$

$$29. \sqrt[3]{v} \cdot \sqrt{v} = v^{\frac{1}{3}} \cdot v^{\frac{1}{2}} = v^{\frac{2}{6} + \frac{3}{6}} = v^{\frac{5}{6}}$$

$$= \sqrt[6]{v^5}$$

$$30. \sqrt[4]{r^3} \cdot \sqrt{r} = r^{\frac{3}{4}} \cdot r^{\frac{1}{2}} = r^{\frac{3}{4} + \frac{2}{4}} = r^{\frac{5}{4}}$$

$$= \sqrt[4]{r^5} = \sqrt[4]{r^4 \cdot r} = \boxed{r \sqrt[4]{r}}$$

$$31. \frac{4}{\sqrt[3]{4}} = \frac{4}{4^{\frac{1}{3}}} = 4^{\frac{3}{3} - \frac{1}{3}} = 4^{\frac{2}{3}}$$

$$= \sqrt[3]{16}$$

$$= \sqrt[3]{8 \cdot 2} = \boxed{2 \sqrt[3]{2}}$$

$$32. \frac{\sqrt[3]{7^3}}{\sqrt{7}} = \frac{7^{\frac{3}{3}}}{7^{\frac{1}{2}}} = \frac{7}{7^{\frac{1}{2}}} = 7^{\frac{2}{2} - \frac{1}{2}} = 7^{\frac{1}{2}}$$

$$= \sqrt{7} = \boxed{7}$$

$$33. \sqrt[4]{x^{10}} = x^{\frac{10}{4}} = x^{\frac{5}{2}}$$

$$= \sqrt{x^5}$$

$$= \sqrt{x^4 \cdot x} = \boxed{x^2 \sqrt{x}}$$

$$34. \sqrt[4]{25m^2} = 25^{\frac{1}{4}} \cdot m^{\frac{2}{4}} = (5^2)^{\frac{1}{4}} \cdot m^{\frac{1}{2}} = 5^{\frac{1}{2}} \cdot m^{\frac{1}{2}} = \sqrt{5m}$$

Name: _____

Unit 6: Radical Functions

Date: _____ Per: _____

Homework 2: Rational Exponents

**Directions:** Rewrite each expression in **radical form**. Simplify if needed.

$$1. 28^{\frac{1}{2}} = \sqrt{28}$$

$$= \sqrt{4 \cdot 7}$$

$$= 2\sqrt{7}$$

$$2. 2^{\frac{4}{3}} = \sqrt[3]{2^4}$$

$$= \sqrt[3]{16}$$

$$= \sqrt[3]{8 \cdot 2} = 2\sqrt[3]{2}$$

$$3. x^{\frac{5}{4}} = \sqrt[4]{x^5}$$

$$= \sqrt[4]{x^4 \cdot x}$$

$$= x\sqrt[4]{x}$$

$$4. (256x)^{\frac{1}{4}} = \sqrt[4]{256x}$$

$$= \sqrt[4]{256} \cdot \sqrt[4]{x}$$

$$= 4\sqrt[4]{x}$$

$$5. (mn)^{\frac{7}{2}} = \sqrt{(mn)^7}$$

$$= \sqrt{(mn)^6 \cdot mn}$$

$$= m^3 n^3 \sqrt{mn}$$

$$6. (-2a)^{\frac{3}{2}} = \sqrt{(-2a)^3}$$

$$= \sqrt{-8a^3}$$

$$= 2a\sqrt[3]{2a}$$

Directions: Rewrite each expression in **exponential form**.

$$7. \sqrt[4]{10^3} = 10^{3/4}$$

$$8. \sqrt{3ab} = (3ab)^{1/2}$$

$$= 3^{1/2} a^{1/2} b^{1/2}$$

$$9. (w^5)^{2/3} = (w^5)^{2/3}$$

$$= w^{10/3}$$

$$10. (18x^6y^2)^{1/4} = (18x^6y^2)^{1/4}$$

$$= 18^{1/4} x^{6/4} y^{2/4}$$

Directions: Simplify each expression. Give answers in **radical form**.

$$11. 9^{\frac{1}{2}} \cdot 9^{\frac{5}{2}} = 9^{\frac{6}{2}}$$

$$= 9^3$$

$$= 729$$

$$12. x^{\frac{7}{3}} = x^{\frac{7}{3}}$$

$$= \sqrt[3]{x^7}$$

$$= x\sqrt[3]{x^2}$$

$$13. (28^{\frac{3}{5}})^{\frac{5}{6}} = 28^{15/30}$$

$$= 28^{1/2}$$

$$= \sqrt{28}$$

$$= \sqrt{4 \cdot 7} = 2\sqrt{7}$$

$$14. (-64)^{\frac{1}{3}} = \sqrt[3]{-64}$$

$$= -4$$

$$15. 45^{\frac{3}{2}} = 45^{1/2}$$

$$= \sqrt{45}$$

$$= \sqrt{9 \cdot 5}$$

$$= 3\sqrt{5}$$

$$16. 2\left(\frac{48}{3}\right)^{-\frac{1}{4}} = 2\left(\frac{1}{16}\right)^{1/4}$$

$$= 2\sqrt[4]{\frac{1}{16}}$$

$$= 2\left(\frac{1}{2}\right) = 1$$

$$17. p^{\frac{3}{2}} = p^{3/2}$$

$$= \sqrt{p^3}$$

$$= \sqrt{p^2 \cdot p} = p\sqrt{p}$$

$$18. \frac{\sqrt[3]{24^4}}{24} = \frac{24^{4/3}}{24} = 24^{1/3}$$

$$= \sqrt[3]{24}$$

$$= \sqrt[3]{8 \cdot 3} = 2\sqrt[3]{3}$$

$$19. \frac{m}{\sqrt[4]{m}} = \frac{m}{m^{1/4}} = m^{3/4}$$

$$= \sqrt[4]{m^3}$$

$$20. \frac{16}{\sqrt[4]{16}} = \frac{16}{16^{1/4}} = 16^{3/4}$$

$$= \sqrt[4]{16^3}$$

$$= 8$$

$$21. \sqrt[4]{a^2b^{14}} = a^{2/4} b^{14/4}$$

$$= a^{1/2} b^{7/2}$$

$$= \sqrt{a} b^{\sqrt{7}}$$

$$= \sqrt{b^6} \sqrt{ab} = b^3 \sqrt{ab}$$

$$22. \sqrt[4]{36w^6} = (36w^6)^{1/4}$$

$$= (6^2w^6)^{1/4}$$

$$= 6^{1/2} w^{3/2}$$

$$= \sqrt{6} w^{\sqrt{3}} = \sqrt{w^2} \sqrt{6w} = w\sqrt{6w}$$

Name: _____

Algebra 2

Date: _____ Per: _____

Unit 6: Radical Functions

Quiz 6-1: Simplifying Radicals & Rational Exponents

Write each expression in simplest radical form.

1. $\sqrt{245}$

$$\sqrt{49} \sqrt{5}$$
$$7\sqrt{5}$$

2. $6\sqrt[3]{-72}$

$$6\sqrt[3]{-8} \sqrt[3]{9}$$
$$6 \cdot (-2) \sqrt[3]{9}$$
$$-12\sqrt[3]{9}$$

3. $-\sqrt[4]{486}$

$$-\sqrt[4]{81} \sqrt[4]{6}$$
$$-3\sqrt[4]{6}$$

4. $\sqrt[4]{144m^7}$

$$\sqrt[4]{16m^4} \sqrt[4]{9m^3}$$
$$2m\sqrt[4]{9m^3}$$

5. $\sqrt{600p^9q^{16}}$

$$\sqrt{100p^8q^{16}} \sqrt{6p}$$
$$10p^4q^8\sqrt{6p}$$

6. $\sqrt[3]{250}$

$$\sqrt[3]{125} \sqrt[3]{2}$$
$$5\sqrt[3]{2}$$

7. $2m\sqrt[4]{9m^3}$

8. $10p^4q^8\sqrt{6p}$

9. $5a^5\sqrt[3]{2b^2}$

10. $14\frac{1}{3}x\frac{7}{3}y\frac{4}{3}$

11. $2\frac{1}{6}c\frac{1}{3}d\frac{4}{3}$

12. $4\sqrt[3]{4}$

13. $81w^2\sqrt{w}$

14. $\sqrt[4]{k}$

15. c^8

16. $r^{12}\sqrt[4]{r}$

17. $y^3\sqrt[3]{y^2}$

Write each expression in exponential form.

7. $\sqrt[3]{14x^7y^4}$

$$(14x^7y^4)^{\frac{1}{3}}$$

8. $\sqrt[4]{2c^2d^8}$

$$(2c^2d^8)^{\frac{1}{4}}$$

Simplify. Write each expression in simplest radical form.

9. $4^3 \cdot 4^{-\frac{5}{3}}$

$$= 4^{\frac{4}{3}}$$

10. $(3w^{\frac{5}{8}})^4$

$$= 3^4 w^{\frac{20}{8}}$$

$$= 81 w^{\frac{5}{2}}$$

$$= 81 \sqrt{w^5}$$

$$= 81 \sqrt{w^4} \sqrt{w}$$

11. $(c^2)^{\frac{7}{4}} \cdot (c^2)^{\frac{9}{4}}$

$$c^{\frac{7}{2}} \cdot c^{\frac{9}{2}} = c^{\frac{16}{2}}$$
$$= c^8$$

12. $(\sqrt{r} \cdot \sqrt[4]{r^5})^7$

$$(r^{\frac{1}{2}})^7 \cdot (r^{\frac{5}{4}})^7$$
$$r^{\frac{7}{2}} \cdot r^{\frac{35}{4}}$$
$$r^{\frac{49}{4}} = \sqrt[4]{r^{48}} \cdot \sqrt[4]{r}$$

13. $\frac{\sqrt[3]{y^{17}}}{y^2}$

$$= \frac{y^{\frac{17}{3}}}{y^2} = y^{\frac{11}{3}}$$
$$= \sqrt[3]{y^{11}} = \sqrt[3]{y^9} \sqrt[3]{y^2}$$

Main Ideas/Questions	Notes/Examples	
ADDING & SUBTRACTING Radicals	① SIMPLY all radicals.	
	② Identify radicals with the SAME INDEX and SAME RADICAND . Only these can be combined!	
	③ For common radicals, add/subtract the coefficients and KEEP THE COMMON RADICAL .	
	1. $3\sqrt{27} - 2\sqrt{12}$ $3\sqrt{9 \cdot 3} - 2\sqrt{4 \cdot 3}$ $3 \cdot 3\sqrt{3} - 2 \cdot 2\sqrt{3}$ $9\sqrt{3} - 4\sqrt{3} = \boxed{5\sqrt{3}}$	2. $-2\sqrt[3]{2} + 7\sqrt[3]{8}$ $-2\sqrt[3]{2} + 7 \cdot \sqrt[3]{(-2) \cdot (-2) \cdot (-2)}$ $-2\sqrt[3]{2} + 7 \cdot (-2)\sqrt[3]{2}$ $-2\sqrt[3]{2} - 14\sqrt[3]{2} = \boxed{-16\sqrt[3]{2}}$
	3. $7\sqrt[4]{48} - 2\sqrt[4]{3} + 3\sqrt[4]{72}$ $7\sqrt[4]{16 \cdot 3} - 2\sqrt[4]{3} + 3\sqrt[4]{9 \cdot 8}$ $7 \cdot 2\sqrt[4]{3} - 2\sqrt[4]{3} + 3 \cdot 2\sqrt[4]{3}$ $14\sqrt[4]{3} - 2\sqrt[4]{3} + 6\sqrt[4]{3}$ $= \boxed{18\sqrt[4]{3}}$	4. $10\sqrt{28} + 4\sqrt{56} - 4\sqrt{175}$ $10\sqrt{4 \cdot 7} + 4\sqrt{4 \cdot 14} - 4\sqrt{25 \cdot 7}$ $10 \cdot 2\sqrt{7} + 4 \cdot 2\sqrt{7} - 4 \cdot 5\sqrt{7}$ $20\sqrt{7} + 8\sqrt{7} - 20\sqrt{7}$ $20\sqrt{7} - 2\sqrt{7} = \boxed{18\sqrt{7}}$
	5. $3x^4y^2 - 2x^2y\sqrt{2}$ $3x^4y^2 - 2x^2y\sqrt{2}$ $\boxed{3x^4y^2 - 2x^2y\sqrt{2}}$	6. $\sqrt[3]{-40a^7} + 2a^2 \cdot \sqrt[3]{135a^4}$ $\sqrt[3]{-8a^6 \cdot 5a} + 2a^2 \cdot \sqrt[3]{27a^3 \cdot 5a}$ $-2a^2 \sqrt[3]{5a} + 2a^2 \cdot 3a \sqrt[3]{5a}$ $= \boxed{-2a^2 \sqrt[3]{5a} + 6a^3 \sqrt[3]{5a}}$
MULTIPLYING Radicals	① Simplify coefficients, then use the PRODUCT RULE : $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{a \cdot b}$	
	② SIMPLIFY the resulting radical.	
	7. $\sqrt{27} \cdot \sqrt{5} = \sqrt{135}$ $= \sqrt{9 \cdot 15}$ $= \boxed{3\sqrt{15}}$	8. $3\sqrt{10} \cdot -2\sqrt{18} = -6\sqrt{180}$ $= -6\sqrt{36 \cdot 5}$ $= -6 \cdot 6\sqrt{5}$ $= \boxed{-36\sqrt{5}}$
	9. $2\sqrt[3]{9} \cdot 5\sqrt[3]{-24} = 10\sqrt[3]{-216}$ $= 10 \cdot -6$ $= \boxed{-60}$	10. $-3\sqrt[4]{64} \cdot -\sqrt[4]{8} = 3\sqrt[4]{512}$ $= 3\sqrt[4]{256 \cdot 2}$ $= 3 \cdot 4\sqrt[4]{2} = \boxed{12\sqrt[4]{2}}$

BINOMIAL Examples

$$\begin{aligned}
 11. \sqrt{6x^4} \cdot 5\sqrt{8x^5} \\
 5\sqrt{48x^9} &= 5\sqrt{16x^8} \sqrt{3x} \\
 &= 5 \cdot 4x^4 \sqrt{3x} \\
 &= \boxed{20x^4 \sqrt{3x}}
 \end{aligned}$$

$$\begin{aligned}
 12. \sqrt[3]{54m^8} \cdot \sqrt[3]{5m^4} \\
 \sqrt[3]{270m^{12}} &= \sqrt[3]{27m^{12}} \sqrt[3]{10} \\
 &= \boxed{3m^4 \sqrt[3]{10}}
 \end{aligned}$$

$$\begin{aligned}
 13. \sqrt[3]{-3a^7b^4} \cdot \sqrt[3]{36a^6b^2} \\
 \sqrt[3]{-108a^{13}b^6} \\
 = \sqrt[3]{-27a^{12}b^6} \sqrt[3]{4a} \\
 = \boxed{-3a^4b^2 \sqrt[3]{4a}}
 \end{aligned}$$

$$\begin{aligned}
 14. 2\sqrt[4]{p^2q} \cdot 7\sqrt[4]{p^3q} \\
 14\sqrt[4]{p^5q^2} \\
 = 14\sqrt[4]{p^4q^2} \sqrt[4]{p} \\
 = \boxed{14q^2 \sqrt[4]{p}}
 \end{aligned}$$

$$\begin{aligned}
 15. \sqrt{10}(5\sqrt{5} - 2\sqrt{2}) \\
 5\sqrt{50} - 2\sqrt{20} \\
 = 5\sqrt{25} \sqrt{2} - 2\sqrt{4} \sqrt{5} \\
 = 5 \cdot 5 \sqrt{2} - 2 \cdot 2 \sqrt{5} \\
 = \boxed{25\sqrt{2} - 4\sqrt{5}}
 \end{aligned}$$

$$\begin{aligned}
 16. (3\sqrt{10} - 5)(3 - \sqrt{10}) \\
 9\sqrt{10} - 3\sqrt{100} - 15 + 5\sqrt{10} \\
 = 24\sqrt{10} - 30 - 15 + 10 \\
 = \boxed{34 - 11\sqrt{10}}
 \end{aligned}$$

$$\begin{aligned}
 17. (6\sqrt{6} - 5\sqrt{2})(\sqrt{2}) \\
 6\sqrt{12} - 5\sqrt{4} \\
 = 6 \cdot 2\sqrt{3} - 5 \cdot 2 \\
 = 12\sqrt{3} - 10 \\
 = \boxed{12\sqrt{3} - 10}
 \end{aligned}$$

$$\begin{aligned}
 18. (4 - \sqrt{5})^2 \\
 (4 - \sqrt{5})(4 - \sqrt{5}) \\
 = 16 - 4\sqrt{5} - 4\sqrt{5} + \sqrt{25} \\
 = 16 - 8\sqrt{5} + 5 \\
 = \boxed{21 - 8\sqrt{5}}
 \end{aligned}$$

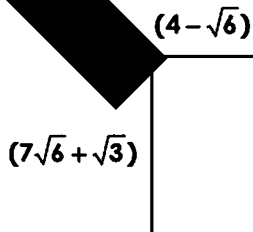
$$\begin{aligned}
 19. \sqrt{3k}(\sqrt{15k^2} - 2\sqrt{15k^2}) \\
 \sqrt{3k} \cdot \sqrt{15k^2} - 2\sqrt{45k^3} \\
 = \sqrt{45k^3} - 2\sqrt{45k^3} \\
 = \sqrt{45k^3} - 2 \cdot 3k\sqrt{5k} \\
 = \boxed{6k - 6k\sqrt{5k}}
 \end{aligned}$$

$$\begin{aligned}
 20. (\sqrt{x} - \sqrt{8})(\sqrt{x} + \sqrt{2}) \\
 \sqrt{x^2} + \sqrt{2x} - \sqrt{8x} - \sqrt{16} \\
 = x + \sqrt{2x} - \sqrt{4} \sqrt{2x} - 4 \\
 = x + \sqrt{2x} - 2\sqrt{2x} - 4 \\
 = \boxed{x - \sqrt{2x} - 4}
 \end{aligned}$$

21. Find the area and perimeter of the rectangle shown to the left.

$$\begin{aligned}
 \text{Area: } (4 - \sqrt{6})(7\sqrt{6} + \sqrt{3}) \\
 = 28\sqrt{6} + 4\sqrt{3} - 7\sqrt{36} - \sqrt{18} \\
 = 28\sqrt{6} + 4\sqrt{3} - 7 \cdot 6 - \sqrt{9} \sqrt{2} \\
 = \boxed{28\sqrt{6} + 4\sqrt{3} - 42 - 3\sqrt{2}}
 \end{aligned}$$

$$\begin{aligned}
 \text{Perimeter: } 2(4 - \sqrt{6}) + 2(7\sqrt{6} + \sqrt{3}) \\
 = 8 - 2\sqrt{6} + 14\sqrt{6} + 2\sqrt{3} \\
 = \boxed{8 + 12\sqrt{6} + 2\sqrt{3}}
 \end{aligned}$$



Name: _____

Unit 6: Radical Functions

Date: _____ Per: _____

Homework 3: Add/Subtract/Multiply Radicals

Directions: Simplify each expression below.

1. $7\sqrt[3]{3} + 2\sqrt[3]{192}$

$7\sqrt[3]{3} + 2\sqrt[3]{64 \cdot 3}$

$7\sqrt[3]{3} + 2 \cdot 4\sqrt[3]{3}$

$7\sqrt[3]{3} + 8\sqrt[3]{3} = \boxed{15\sqrt[3]{3}}$

2. $10\sqrt{7} - \sqrt{28} + 6\sqrt{180}$

$10\sqrt{7} - \sqrt{4 \cdot 7} + 6\sqrt{36 \cdot 5}$

$10\sqrt{7} - 2\sqrt{7} + 36\sqrt{5}$

$= \boxed{8\sqrt{7} + 36\sqrt{5}}$

3. $8\sqrt[4]{48} - 5\sqrt{90} + 9\sqrt[4]{3}$

$8\sqrt[4]{16 \cdot 3} - 5\sqrt{9 \cdot 10} + 9\sqrt[4]{3}$

$8 \cdot 2\sqrt[4]{3} - 5 \cdot 3\sqrt{10} + 9\sqrt[4]{3}$

$= \boxed{25\sqrt[4]{3} - 15\sqrt{10}}$

4. $5\sqrt[3]{32x^3y^4} - \sqrt[3]{4y}$

$5\sqrt[3]{8x^3 \cdot 4y}$

$5 \cdot 2x\sqrt[3]{4y}$

$= \boxed{10x\sqrt[3]{4y}}$

5. $15\sqrt{2} - 2\sqrt{20}$

$-30\sqrt{40}$

$-30\sqrt{4 \cdot 10}$

$-30 \cdot 2\sqrt{10} = \boxed{-60\sqrt{10}}$

6. $4\sqrt[3]{-16}$

$28\sqrt[3]{-16}$

$28\sqrt[3]{-8 \cdot 2}$

$28 \cdot -2\sqrt[3]{2} = \boxed{-168\sqrt[3]{2}}$

7. $3\sqrt{18w^7} \cdot 10\sqrt{4w^9}$

$30\sqrt{72w^{16}}$

$30\sqrt{36 \cdot 2 \cdot w^{16}}$

$30 \cdot 6w^4\sqrt{2} = \boxed{180w^4\sqrt{2}}$

8. $\sqrt{8a^3b} \cdot \sqrt[4]{10a^2b^7}$

$\sqrt[4]{80a^5b^8} = \sqrt[4]{16a^4b^8} \sqrt[4]{5a}$

$= \boxed{2ab^2\sqrt[4]{5a}}$

9. $2\sqrt{3}(\sqrt{2} - \sqrt{18})$

$2\sqrt{3} \cdot \sqrt{2} - 2\sqrt{3} \cdot \sqrt{18}$

$2\sqrt{6} - 10\sqrt{6}$

$18 - 16 \cdot 3\sqrt{2} = \boxed{18 - 48\sqrt{2}}$

10. $(5\sqrt{5} - 2)(\sqrt{5} - 1)$

$5\sqrt{25} - 5\sqrt{5} - 2\sqrt{5} + 2$

$5 \cdot 5 - 7\sqrt{5} + 2$

$\boxed{27 - 7\sqrt{5}}$

11. $(100 - 30\sqrt{7})(10 - 3\sqrt{7})$

$1000 - 300\sqrt{7} + 300\sqrt{7} - 9 \cdot 49$

$1000 - 9 \cdot 7$

$1000 - 63 = \boxed{937}$

12. $(3 - 2\sqrt{k})^2$

$(3 - 2\sqrt{k})(3 - 2\sqrt{k})$

$9 - 6\sqrt{k} - 6\sqrt{k} + 4\sqrt{k^2}$

$\boxed{9 - 12\sqrt{k} + 4k}$

$$\begin{aligned}
 11. \frac{3\sqrt{12} \cdot \sqrt{7}}{4\sqrt{7} \cdot \sqrt{7}} &= \frac{3\sqrt{84}}{4\sqrt{49}} \\
 &= \frac{3\sqrt{4} \sqrt{21}}{4 \cdot 7} \\
 &= \frac{6\sqrt{21}}{28} = \boxed{\frac{3\sqrt{21}}{14}}
 \end{aligned}$$

$$\begin{aligned}
 12. \frac{\sqrt{32a^5}}{\sqrt{3a}} &= \frac{\sqrt{32a^4} \cdot \sqrt{a}}{\sqrt{3} \cdot \sqrt{a}} \\
 &= \frac{\sqrt{96a^4}}{\sqrt{3}} \\
 &= \frac{\sqrt{16a^4} \sqrt{6}}{\sqrt{3}} \\
 &= \frac{4a^2 \sqrt{6}}{\sqrt{3}} = \boxed{\frac{4a^2 \sqrt{2}}{1}}
 \end{aligned}$$

$$\begin{aligned}
 13. \frac{(\sqrt{8}-\sqrt{2}) \cdot \sqrt{2}}{\sqrt{2} \cdot \sqrt{2}} \\
 &= \frac{\sqrt{16} - \sqrt{4}}{\sqrt{4}} \\
 &= \frac{4-2}{2} = \boxed{1}
 \end{aligned}$$

$$\begin{aligned}
 14. \frac{(3\sqrt{2} + \sqrt{18}) \cdot \sqrt{2}}{\sqrt{2} \cdot \sqrt{2}} \\
 &= \frac{3\sqrt{4} + \sqrt{12}}{2} \\
 &= \frac{6 + 2\sqrt{3}}{2} = \boxed{\frac{\sqrt{6} + \sqrt{2}}{2}}
 \end{aligned}$$

BINOMIAL
Denominators

$$\begin{aligned}
 15. \frac{(4)(4-\sqrt{2})}{(4+\sqrt{2})(4-\sqrt{2})} \\
 &= \frac{16-4\sqrt{2}}{16-4\sqrt{2} \cdot \sqrt{2}} \\
 &= \frac{16-4\sqrt{2}}{16-8} = \boxed{\frac{16-4\sqrt{2}}{8}}
 \end{aligned}$$

$$\begin{aligned}
 \frac{(8)(6+\sqrt{5})}{(\sqrt{5})(6+\sqrt{5})} \\
 &= \frac{48+8\sqrt{5}}{36+6\sqrt{5}-6\sqrt{5}-\sqrt{25}} \\
 &= \frac{48+8\sqrt{5}}{36-25} = \boxed{\frac{12+2\sqrt{5}}{31}}
 \end{aligned}$$

$$\begin{aligned}
 17. \frac{(1+\sqrt{3})(1-\sqrt{3})}{(1-4\sqrt{2})(1+4\sqrt{2})} \\
 &= \frac{1-3}{1-16\sqrt{2} \cdot \sqrt{2}} \\
 &= \frac{-2}{1-32} = \boxed{\frac{\sqrt{3}-4\sqrt{6}}{31}}
 \end{aligned}$$

$$\begin{aligned}
 18. \frac{(5-\sqrt{5})(\sqrt{5}-\sqrt{3})}{(\sqrt{5}+\sqrt{3})(\sqrt{5}-\sqrt{3})} \\
 &= \frac{5\sqrt{5}-5\sqrt{3}-\sqrt{25}+\sqrt{15}}{\sqrt{25}-\sqrt{15}+\sqrt{15}-\sqrt{9}} \\
 &= \frac{5\sqrt{5}-5\sqrt{3}-5+\sqrt{15}}{2}
 \end{aligned}$$

$$\begin{aligned}
 19. \frac{(3+\sqrt{7})(2-2\sqrt{7})}{(2+2\sqrt{7})(2-2\sqrt{7})} \\
 &= \frac{6-6\sqrt{7}+2\sqrt{7}-2\sqrt{49}}{4-4\sqrt{7}+4\sqrt{7}-4\sqrt{49}} \\
 &= \frac{6-4\sqrt{7}-14}{4-28} = \frac{-8-4\sqrt{7}}{-24} \\
 &= \boxed{\frac{2+\sqrt{7}}{6}}
 \end{aligned}$$

$$\begin{aligned}
 20. \frac{(8-\sqrt{6})(3+4\sqrt{6})}{(3-4\sqrt{6})(3+4\sqrt{6})} \\
 &= \frac{24+32\sqrt{6}-3\sqrt{6}-4\sqrt{36}}{9+12\sqrt{6}-12\sqrt{6}-16\sqrt{36}} \\
 &= \frac{24+29\sqrt{6}-24}{9-96} \\
 &= \frac{29\sqrt{6}}{-87} = \boxed{-\frac{\sqrt{6}}{3}}
 \end{aligned}$$

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Unit 6: Radical Functions

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Homework 4: Dividing Radicals

**** This is a 2-page document! ******Directions:** Simplify each expression below.

$$1. \frac{20\sqrt{270}}{4\sqrt{3}} = 5\sqrt{90}$$

$$= 5\sqrt{9}\sqrt{10}$$

$$= \boxed{15\sqrt{10}}$$

$$2. \frac{6\sqrt[4]{336}}{2\sqrt[4]{7}} = 3\sqrt[4]{48}$$

$$= 3\sqrt[4]{16\sqrt[4]{3}}$$

$$= 3 \cdot 2\sqrt[4]{3}$$

$$= \boxed{6\sqrt[4]{3}}$$

$$3. \frac{\sqrt[3]{a^{10}b^{16}}}{\sqrt[3]{a^5b^4}} = \sqrt[3]{a^5b^{12}}$$

$$= \sqrt[3]{a^3b^{12}} \sqrt[3]{a^2}$$

$$= \boxed{ab^4\sqrt[3]{a^2}}$$

$$4. \frac{24\sqrt{225x^{10}}}{3x^4} = \frac{6\sqrt{45x^{10}}}{x^4}$$

$$= \frac{6 \cdot 3x^5\sqrt{5}}{x^4}$$

$$= \boxed{18x\sqrt{5}}$$

$$5. \frac{\sqrt[3]{-324}}{2} = \frac{\sqrt[3]{-162}}{2}$$

$$= \frac{\sqrt[3]{-27} \sqrt[3]{6}}{2}$$

$$= \boxed{-\frac{3\sqrt[3]{6}}{2}}$$

$$\frac{2m^7}{5n^4} = \frac{\sqrt[4]{m^4} \sqrt[4]{2m^3}}{5n}$$

$$= \boxed{\frac{m\sqrt[4]{2m^3}}{5n}}$$

$$7. \frac{\sqrt{\frac{3}{20}}}{\sqrt{\frac{1}{5}}} = \frac{\sqrt{60}}{\sqrt{4}\sqrt{15}}$$

$$= \frac{20}{2\sqrt{15}} = \boxed{\frac{\sqrt{15}}{10}}$$

$$8. \frac{\sqrt{8}}{2\sqrt{14}} = \frac{\sqrt{4}}{2\sqrt{7}} = \frac{2}{2\sqrt{7}}$$

$$= \frac{1}{\sqrt{7}} \cdot \frac{\sqrt{7}}{\sqrt{7}}$$

$$= \boxed{\frac{\sqrt{7}}{7}}$$

$$9. \frac{\sqrt{\frac{5}{6}}}{\sqrt{\frac{1}{6}}} = \frac{3\sqrt{40}}{8\sqrt{25}}$$

$$= \frac{3\sqrt{4}\sqrt{10}}{40}$$

$$= \frac{6\sqrt{10}}{40} = \boxed{\frac{3\sqrt{10}}{20}}$$

$$10. \frac{\sqrt{\frac{18}{30}}}{\sqrt{\frac{1}{15}}} = \frac{\sqrt{9}}{\sqrt{15}} = \frac{3}{\sqrt{15} \cdot \sqrt{15}}$$

$$= \frac{3\sqrt{15}}{15}$$

$$= \boxed{\frac{\sqrt{15}}{5}}$$

$$\begin{aligned}
 11. \frac{\sqrt{5n}}{2\sqrt{15n^4}} &= \frac{1}{2\sqrt{3n^3}} \\
 &= \frac{1}{2\sqrt{n^2}\sqrt{3n}} \\
 &= \frac{1}{2n\sqrt{3n}} \cdot \frac{\sqrt{3n}}{\sqrt{3n}} \\
 &= \frac{\sqrt{3n}}{2n \cdot 3n} = \boxed{\frac{\sqrt{3n}}{6n^2}}
 \end{aligned}$$

$$\begin{aligned}
 12. \frac{4y}{\sqrt{2x^4y}} &= \frac{4y}{\sqrt{x^4}\sqrt{2y}} = \frac{4y}{x^2\sqrt{2y}} \cdot \frac{\sqrt{2y}}{\sqrt{2y}} \\
 &= \frac{4y\sqrt{2y}}{x^2 \cdot 2y} \\
 &= \frac{4y}{2x^2} = \boxed{\frac{2y}{x^2}}
 \end{aligned}$$

$$\begin{aligned}
 13. \frac{\sqrt{5}-\sqrt{6}}{\sqrt{18}} &= \frac{\sqrt{5}-\sqrt{6}}{\sqrt{9}\sqrt{2}} = \frac{(\sqrt{5}-\sqrt{6})(\sqrt{2})}{(3\sqrt{2})(\sqrt{2})} \\
 &= \frac{\sqrt{10}-\sqrt{12}}{3\sqrt{4}} \\
 &= \frac{\sqrt{10}-\sqrt{4}\sqrt{3}}{6} \\
 &= \boxed{\frac{\sqrt{10}-2\sqrt{3}}{6}}
 \end{aligned}$$

$$\begin{aligned}
 14. \frac{4-\sqrt{2}}{2\sqrt{8}} &= \frac{4-\sqrt{2}}{2\sqrt{4}\sqrt{2}} = \frac{(4-\sqrt{2})(\sqrt{2})}{(4\sqrt{2})(\sqrt{2})} \\
 &= \frac{4\sqrt{2}-2}{8} \\
 &= \boxed{\frac{-1+2\sqrt{2}}{4}}
 \end{aligned}$$

$$\begin{aligned}
 15. \frac{12}{(4-\sqrt{3})(4+\sqrt{3})} &= \frac{12}{16+4\sqrt{3}-4\sqrt{3}-12} \\
 &= \frac{12}{4} = \boxed{3}
 \end{aligned}$$

$$\begin{aligned}
 17. \frac{(2-\sqrt{3})(6+\sqrt{3})}{(6-\sqrt{3})(6+\sqrt{3})} &= \frac{12\sqrt{3}+2\sqrt{9}-18-3\sqrt{3}}{36+6\sqrt{3}-6\sqrt{3}-9} \\
 &= \frac{9\sqrt{3}+6-18}{33} \\
 &= \frac{-12+9\sqrt{3}}{33} = \boxed{\frac{-4+3\sqrt{3}}{11}}
 \end{aligned}$$

$$\begin{aligned}
 &\frac{\sqrt{2}(6-\sqrt{2})}{(6-\sqrt{2})(6-\sqrt{2})} \\
 &= \frac{6\sqrt{2}-\sqrt{4}}{36-6\sqrt{2}+6\sqrt{2}-4} \\
 &= \frac{6\sqrt{2}-2}{34} \\
 &= \boxed{\frac{-1+3\sqrt{2}}{17}}
 \end{aligned}$$

$$\begin{aligned}
 18. \frac{(6+5\sqrt{2})(4-3\sqrt{2})}{(4+3\sqrt{2})(4-3\sqrt{2})} &= \frac{24-18\sqrt{2}+20\sqrt{2}-15\sqrt{4}}{16-12\sqrt{2}+12\sqrt{2}-9\sqrt{4}} \\
 &= \frac{24+2\sqrt{2}-30}{16-18} \\
 &= \frac{-6+2\sqrt{2}}{-2} = \boxed{3-\sqrt{2}}
 \end{aligned}$$

REVIEW: Radicals & Rational Exponents

SIMPLIFYING RADICALS

Write each expression in simplest radical form.

1. $\sqrt{300m^4n^{25}}$

$$\sqrt{100m^4n^{24}} \sqrt{3n}$$

$$= \boxed{10m^2n^{12}\sqrt{3n}}$$

2. $\sqrt[3]{-72a^9b^2}$

$$\sqrt[3]{-8a^9} \sqrt[3]{9b^2}$$

$$= \boxed{-2a^3\sqrt[3]{9b^2}}$$

3. $\sqrt[4]{81m^{11}n^{20}}$

$$\sqrt[4]{81m^8n^{20}} \sqrt[4]{m^3}$$

$$= \boxed{3m^2n^5\sqrt[4]{m^3}}$$

OPERATIONS WITH RADICALS

Perform the indicated operation(s). Write each answer in simplest radical form.

4. $4\sqrt{98} + \sqrt{150} - 2\sqrt{32}$

$$4\sqrt{49}\sqrt{2} + \sqrt{25}\sqrt{6} - 2\sqrt{16}\sqrt{2}$$

$$= 28\sqrt{2} + 5\sqrt{6} - 8\sqrt{2}$$

$$= \boxed{20\sqrt{2} + 5\sqrt{6}}$$

5. $10\sqrt[4]{5} + \sqrt[4]{320} - \sqrt[3]{32}$

$$10\sqrt[4]{5} + \sqrt[4]{64\sqrt[4]{5}} - \sqrt[3]{8\sqrt[3]{5}}$$

$$= 10\sqrt[4]{5} + 14\sqrt[4]{5} - 2\sqrt[3]{5}$$

$$= \boxed{24\sqrt[4]{5} - 2\sqrt[3]{5}}$$

6. $3\sqrt[4]{8} \cdot 7\sqrt[4]{32}$

$$21\sqrt[4]{256}$$

$$= 21 \cdot 4 = \boxed{84}$$

7. $\sqrt[3]{-6x^5}$

$$\sqrt[3]{-10x^3} \sqrt[3]{-6x^2} = \sqrt[3]{-2x^3} \sqrt[3]{4}$$

$$= \boxed{-3x\sqrt[3]{4}}$$

8. $\sqrt{3}(2 - \sqrt{24})$

$$2\sqrt{3} - \sqrt{3}\sqrt{24}$$

$$= 2\sqrt{3} - \sqrt{72}$$

$$= \boxed{2\sqrt{3} - 6\sqrt{2}}$$

9. $(4 + 3\sqrt{5}) \cdot (-2 + \sqrt{5})$

$$-8 + 4\sqrt{5} - 6\sqrt{5} + 3\sqrt{25}$$

$$= -8 - 2\sqrt{5} + 15$$

$$= \boxed{7 - 2\sqrt{5}}$$

10. $(4 - 7\sqrt{3})(1 + \sqrt{3})$

$$4 + 4\sqrt{3} - 7\sqrt{3} - 49\sqrt{9}$$

$$= 4 - 3\sqrt{3} - 147$$

$$= \boxed{-146 - 3\sqrt{3}}$$

11. $(3 + \sqrt{6})^2$

$$(3 + \sqrt{6})(3 + \sqrt{6})$$

$$= 9 + 3\sqrt{6} + 3\sqrt{6} + \sqrt{36}$$

$$= 9 + 6\sqrt{6} + 6 = \boxed{15 + 6\sqrt{6}}$$

12. $-\frac{5}{9}\sqrt{52}$

$$= -5\sqrt{52}$$

$$= -5\sqrt{4}\sqrt{13}$$

$$= \boxed{-10\sqrt{13}}$$

13. $\frac{\sqrt[4]{r^{20}s^9}}{\sqrt[4]{r^3s}}$

$$\sqrt[4]{r^{17}s^8}$$

$$= \sqrt[4]{r^{16}s^8} \sqrt[4]{r}$$

$$= \boxed{r^4s^2\sqrt[4]{r}}$$

14. $\sqrt{\frac{50}{81}} = \frac{\sqrt{25} \sqrt{2}}{\sqrt{81}} = \boxed{\frac{5\sqrt{2}}{9}}$	15. $\frac{\sqrt[4]{3a}}{\sqrt[4]{16b^8}} = \boxed{\frac{\sqrt[4]{3a}}{2b^2}}$
16. $\frac{\sqrt{6}}{4\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{18}}{4\sqrt{9}} = \frac{\sqrt{9} \sqrt{2}}{4\sqrt{9}} = \boxed{\frac{\sqrt{2}}{4}}$	17. $\sqrt{\frac{8x}{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{40x}}{\sqrt{25}} = \frac{4\sqrt{10x}}{5}$
18. $\frac{(2-\sqrt{12})\sqrt{3}}{\sqrt{3} \cdot \sqrt{3}} = \frac{2\sqrt{3} - \sqrt{36}}{\sqrt{9}} = \boxed{\frac{2\sqrt{3}-6}{3}}$	19. $\frac{3+\sqrt{3}}{4\sqrt{12}} = \frac{3+\sqrt{3}}{4\sqrt{3}} = \frac{(3+\sqrt{3})(\sqrt{3})}{(8\sqrt{3})} = \frac{\sqrt{3}+\sqrt{9}}{8\sqrt{3}} = \boxed{\frac{\sqrt{3}+1}{8}}$
20. $\frac{(5-4\sqrt{2})(4+3\sqrt{2})}{(4-3\sqrt{2})(4+3\sqrt{2})} = \frac{20+15\sqrt{2}-16\sqrt{2}-12\sqrt{4}}{16+12\sqrt{2}-12\sqrt{2}-9\sqrt{4}} = \frac{20-\sqrt{2}-24}{16-18} = \frac{-4-\sqrt{2}}{-2} = \boxed{\frac{4+\sqrt{2}}{2}}$	21. $\frac{(\sqrt{3}-5)(\sqrt{3}-5)}{(\sqrt{3}-5)(\sqrt{3}-5)} = \frac{2\sqrt{3}\sqrt{9}-15\sqrt{3}}{\sqrt{9}\sqrt{3}+5\sqrt{3}-25} = \frac{-13\sqrt{3}-10+9}{-22} = \frac{-1-13\sqrt{3}}{-22} = \boxed{\frac{1+13\sqrt{3}}{22}}$

RATIONAL EXPONENTS

Simplify each expression. Write the final answer in simplest radical form.

22. $x^{\frac{2}{3}} \cdot x^{\frac{11}{3}} = 3\sqrt[3]{x^{12}} = \boxed{x^4}$	23. $32^{\frac{3}{8}} = 32^{\frac{4}{8}} = 32^{\frac{1}{2}} = \sqrt{32} = \sqrt{16} \sqrt{2} = \boxed{4\sqrt{2}}$	24. $n^{\frac{7}{3}} \cdot n^3 = n^{\frac{16}{3}} = \sqrt[3]{n^{16}} = \sqrt[3]{n^{15}} \sqrt[3]{n} = \boxed{n^5 \sqrt[3]{n}}$
25. $\frac{7^{\frac{7}{4}}}{7^{\frac{1}{4}}} = 7^{\frac{6}{4}} = 7^{\frac{3}{2}} = \boxed{7\sqrt{7}}$	26. $(k^{\frac{3}{2}})^{\frac{1}{2}} = k^{\frac{3}{4}} = \sqrt[4]{k^3}$	27. $1296^{-\frac{1}{4}} = \frac{1}{\sqrt[4]{1296}} = \boxed{\frac{1}{6}}$
28. $\frac{125^{\frac{3}{2}}}{125^{\frac{2}{2}}} = \frac{1}{125^{\frac{1}{2}}} = \frac{1}{\sqrt{125}} = \boxed{\frac{1}{5}}$	29. $\frac{\sqrt[4]{3^3}}{\sqrt{3}} = \frac{3^{\frac{3}{4}}}{3^{\frac{1}{2}}} = 3^{\frac{1}{4}} = \sqrt[4]{3}$	30. $\sqrt[4]{10^2 a^{18}} = (10^2 a^{18})^{\frac{1}{4}} = 10^{\frac{1}{2}} a^{\frac{9}{2}} = \boxed{a^4 \sqrt{10a}}$

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Homework 5: Radicals & Rational Exponents

**** This is a 2-page document! ******Directions:** Simplify each expression.

1. $7\sqrt{192m^9n^2}$

$$7\sqrt{64m^8n^2} \sqrt{3m}$$

$$= 7 \cdot 8m^4n \sqrt{3m}$$

$$= \boxed{56m^4n\sqrt{3m}}$$

2. $\sqrt[3]{-135x^6y^{13}}$

$$\sqrt[3]{-27x^6y^{12}} \sqrt[3]{5y}$$

$$= \boxed{-3x^2y^4\sqrt[3]{5y}}$$

3. $\sqrt[4]{1,250a^7b^{24}}$

$$\sqrt[4]{625a^4b^{24}}$$

$$= \boxed{5ab^6}$$

4. $-2\sqrt{27} - \sqrt{18} + 3\sqrt{72}$

$$-2\sqrt{9\sqrt{3}} - \sqrt{9\sqrt{2}} + 3\sqrt{36\sqrt{2}}$$

$$= -6\sqrt{3} - 3\sqrt{2} + 18\sqrt{2}$$

$$= \boxed{-6\sqrt{3} + 15\sqrt{2}}$$

5. $2\sqrt[4]{405} - 3\sqrt[4]{192}$

$$2\sqrt[4]{81\sqrt{5}} - 3\sqrt[4]{64\sqrt{3}}$$

$$= 2\sqrt[4]{5} - 3\sqrt[4]{3}$$

$$= \boxed{-4\sqrt[4]{3}}$$

6. $-3\sqrt{6} \cdot -6\sqrt{8}$

$$18\sqrt{48}$$

$$= 18\sqrt{16\sqrt{3}}$$

$$= 18 \cdot 4\sqrt{3} = \boxed{72\sqrt{3}}$$

7. $5\sqrt[3]{-3m^3w^5}$

$$10\sqrt[3]{-27w^3}\sqrt[3]{4w^2}$$

$$= \boxed{-30w\sqrt[3]{4w^2}}$$

8. $\sqrt{15}(2\sqrt{6} - 7\sqrt{5})$

$$2\sqrt{90} - 7\sqrt{75}$$

$$= 2\sqrt{9\sqrt{10}} - 7\sqrt{25\sqrt{3}}$$

$$= \boxed{6\sqrt{10} - 35\sqrt{3}}$$

9. $(3\sqrt{3} - 2)(4\sqrt{3} + 5)$

$$12\sqrt{9} + 15\sqrt{3} - 8\sqrt{3} - 10$$

$$= 12 \cdot 3 + 7\sqrt{3} - 10$$

$$= 36 + 7\sqrt{3} - 10 = \boxed{26 + 7\sqrt{3}}$$

10. $(1 - 7\sqrt{2})(1 + 7\sqrt{2})$

$$1 - 49 \cdot 2$$

$$1 - 98 = \boxed{-97}$$

11. $(5 + \sqrt{12})^2$

$$(5 + \sqrt{12})(5 + \sqrt{12})$$

$$= 25 + 5\sqrt{12} + 5\sqrt{12} + \sqrt{144}$$

$$= 25 + 10\sqrt{12} + 12$$

$$= 37 + 10\sqrt{4\sqrt{3}} = \boxed{37 + 20\sqrt{3}}$$

12. $\sqrt[5]{-15}$

$$= -4\sqrt{63}$$

$$= -4\sqrt{9\sqrt{7}}$$

$$= \boxed{-12\sqrt{7}}$$

13. $\sqrt{\frac{12}{27}} = \frac{\sqrt{4}}{\sqrt{9}} = \boxed{\frac{2}{3}}$

14. $\frac{8\sqrt[3]{144a^{15}}}{4\sqrt[3]{3a^5}} = 2\sqrt[3]{48a^{10}}$ $= 2\sqrt[3]{8a^9} \sqrt[3]{6a}$ $= 2 \cdot 2a^3 \sqrt[3]{6a}$ $= \boxed{4a^3\sqrt[3]{6a}}$	15. $\frac{5\sqrt[4]{2}}{4\sqrt[4]{162}} = \frac{5}{4\sqrt[4]{81}}$ $= \frac{5}{4 \cdot 3} = \boxed{\frac{5}{12}}$
16. $\frac{6\sqrt{20}}{\sqrt{18}} = \frac{6\sqrt{10}}{\sqrt{9}}$ $= \frac{6\sqrt{10}}{3} = \boxed{2\sqrt{10}}$	17. $\frac{(4+\sqrt{6})\sqrt{6}}{(3\sqrt{6})\sqrt{6}}$ $= \frac{4\sqrt{6} + \sqrt{6} \cdot \sqrt{6}}{3 \cdot 6} = \frac{4\sqrt{6} + 6}{18}$ $= \frac{2\sqrt{6} + 3}{9}$
18. $\frac{7(4-\sqrt{5})}{(4+\sqrt{5})(4-\sqrt{5})}$ $= \frac{28-7\sqrt{5}}{16-4\sqrt{5}+4\sqrt{5}-\sqrt{25}}$ $= \frac{28-7\sqrt{5}}{16-5} = \boxed{\frac{28-7\sqrt{5}}{11}}$	19. $\frac{(5\sqrt{2})(2+\sqrt{2})}{(\sqrt{2})(2+\sqrt{2})}$ $= \frac{10\sqrt{2} + 10}{2 + 2\sqrt{2} + 2\sqrt{2} + 2}$ $= \frac{10\sqrt{2} + 10}{4 + 4\sqrt{2}} = \frac{10\sqrt{2} + 10}{4(1+\sqrt{2})}$ $= \frac{10\sqrt{2} + 10}{4(1+\sqrt{2})} \cdot \frac{1-\sqrt{2}}{1-\sqrt{2}} = \frac{10\sqrt{2} + 10}{4(1-2)} = \frac{10\sqrt{2} + 10}{-4}$ $= -\frac{10\sqrt{2} + 10}{4} = -\frac{5\sqrt{2} + 5}{2} = \boxed{-\frac{5\sqrt{2} + 5}{2}}$
Directions: Use the exponent rules to simplify. Write answers in simplest radical form.	
20. $5r^3 \cdot 2r^{\frac{1}{2}} = 10r^{7\frac{1}{2}}$ $= 10\sqrt{r^7}$ $= 10\sqrt{r^6 r} = \boxed{10r^3\sqrt{r}}$	21. $(3v)^{\frac{2}{3}} \cdot (3v)^{\frac{5}{3}} = (3v)^{\frac{7}{3}}$ $= 3\sqrt[3]{(3v)^7}$ $= 3\sqrt[3]{(3v)^6 (3v)} = \boxed{3v\sqrt[3]{9v^2}}$
22. $\frac{n^{\frac{7}{4}}}{n^{\frac{5}{4}}} = n^{\frac{2}{4}}$ $= n^{\frac{1}{2}}$ $= \boxed{\sqrt{n}}$	
23. $(243x^{\frac{1}{3}})^{\frac{5}{3}} = 243^{\frac{5}{3}} x^{\frac{5}{9}}$ $= 243\sqrt[3]{243} x^{\frac{5}{9}} = \boxed{243\sqrt[3]{243} x^{\frac{5}{9}}}$	24. $\frac{(54^{\frac{1}{2}})^{\frac{5}{3}}}{54^{\frac{1}{2}}} = \frac{54^{\frac{5}{6}}}{54^{\frac{1}{2}}}$ $= 54^{\frac{5}{6} - \frac{1}{2}} = 54^{\frac{1}{3}}$ $= \sqrt[3]{54} = \boxed{3\sqrt[3]{2}}$
25. $(\frac{144}{9})^{\frac{1}{4}} = (\frac{9}{144})^{\frac{1}{4}}$ $= (\frac{1}{16})^{\frac{1}{4}} = \boxed{\frac{1}{2}}$	
26. $12p^{\frac{5}{4}} = 12p^{\frac{9}{4}}$ $= 12\sqrt[4]{p^9}$ $= 12\sqrt[4]{p^8 p} = \boxed{12p^2\sqrt[4]{p}}$	27. $\frac{m^2}{\sqrt[3]{m}} = \frac{m^2}{m^{\frac{1}{3}}} = m^{\frac{5}{3}}$ $= \sqrt[3]{m^5}$ $= \sqrt[3]{m^3 m^2} = \boxed{m\sqrt[3]{m^2}}$
	28. $\sqrt[4]{49y^{22}} = 49^{\frac{1}{4}} y^{\frac{11}{2}}$ $= (7^2)^{\frac{1}{4}} y^{\frac{11}{2}}$ $= 7^{\frac{1}{2}} y^{\frac{11}{2}}$ $= \boxed{7\sqrt{7}y^{\frac{11}{2}}}$

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Algebra 2

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Unit 6: Radical Functions

Quiz 6-2: Radicals (Simplifying & Operations), Rational Exponents**Part I: Radicals** – Simplify. Write all final answers in simplest radical form.

1. $\sqrt[3]{256a^{15}b^2}$

$$\sqrt[3]{64a^{15}} \sqrt[3]{4b^2}$$

2. $8\sqrt[4]{32} - \sqrt[4]{162}$

$$8\sqrt[4]{16}\sqrt[4]{2} - \sqrt[4]{81}\sqrt[4]{2}$$

$$8 \cdot 2\sqrt[4]{2} - 3\sqrt[4]{2}$$

$$16\sqrt[4]{2} - 3\sqrt[4]{2}$$

3. $\sqrt[3]{-3x^2} \cdot \sqrt[3]{24x^5}$

$$\sqrt[3]{-72x^7}$$

$$\sqrt[3]{-8x^6} \sqrt[3]{9x}$$

4. $\sqrt{2}(\sqrt{5} - 6\sqrt{12})$

$$\sqrt{10} - 6\sqrt{24}$$

$$\sqrt{10} - 4\sqrt{6} - 6\sqrt{10}$$

$$\sqrt{10} - 10\sqrt{6} + 2\sqrt{10}$$

5. $(4\sqrt{7} + 3)(6\sqrt{7} - 1)$

$$24\sqrt{49} - 4\sqrt{7} + 18\sqrt{7} - 3$$

$$168 + 14\sqrt{7} - 3$$

6. $(\sqrt{3} + \sqrt{2})(\sqrt{3} + \sqrt{2})$

$$\sqrt{9} + \sqrt{6} + \sqrt{6} + \sqrt{4}$$

$$+ 2\sqrt{6} + 2$$

7. $\frac{-8\sqrt{2}}{2\sqrt{5}}$

$$\frac{-4\sqrt{2}}{\sqrt{5}}$$

$$-\frac{4\sqrt{10}}{5}$$

$$-1.2\sqrt{10}$$

8. $\frac{\sqrt[4]{p^7q^{10}}}{\sqrt[4]{p^2q^2}}$

$$\sqrt[4]{p^5q^8}$$

$$\sqrt[4]{p^4q^8} \sqrt[4]{p}$$

9. $\frac{2}{\sqrt{12}\sqrt{4}\sqrt{3}}$

$$\frac{1}{\sqrt{3} \cdot \sqrt{3}} = \frac{\sqrt{3}}{\sqrt{9}}$$

10. $\frac{10\sqrt{18} \cdot \sqrt{5}}{\sqrt{5} \cdot \sqrt{5}} = \frac{10\sqrt{90}}{\sqrt{25}} = \frac{10\sqrt{9}\sqrt{10}}{5}$
$$= \frac{10 \cdot 3\sqrt{10}}{5}$$

1. $\sqrt[4]{16}$

13

3. $-2\sqrt{10}$

4. $2\sqrt{10} - 12\sqrt{6}$

$$165 + 14\sqrt{7}$$

6. $5 + 2\sqrt{6}$

7. $-8\sqrt{7}$

8. $pq^2\sqrt[4]{p}$

9. $\frac{\sqrt{3}}{3}$

10. $6\sqrt{10}$

$$11. \frac{\sqrt{8m^2}}{\sqrt{6m}} \cdot \frac{\sqrt{4m} \cdot \sqrt{3}}{\sqrt{3} \cdot \sqrt{3}} = \frac{2\sqrt{3m}}{\sqrt{9}}$$

$$12. \frac{(\sqrt{5} + \sqrt{2})\sqrt{2}}{\sqrt{8} \cdot \sqrt{2}} = \frac{\sqrt{10} + \sqrt{4}}{\sqrt{16}}$$

$$11. \frac{2\sqrt{3m}}{3}$$

$$12. \frac{2 + \sqrt{10}}{4}$$

$$13. \frac{24 - \sqrt{11}}{11}$$

$$14. \frac{2\sqrt{2}}{5}$$

$$13. \frac{6(4 - \sqrt{5})}{(4 + \sqrt{5})(4 - \sqrt{5})} = \frac{24 - 6\sqrt{5}}{16 - 4\sqrt{5} + 4\sqrt{5} - \sqrt{25}} = \frac{24 - 6\sqrt{5}}{16 - 5}$$

$$14. \frac{(4 + \sqrt{2})(5\sqrt{2} + 2)}{(5\sqrt{2} - 2)(5\sqrt{2} + 2)} = \frac{20\sqrt{2} + 8 + 5\sqrt{4} + 2}{25\sqrt{4} + 10\sqrt{2} - 10\sqrt{2} - 4} = \frac{22\sqrt{2} + 8 + 10}{50 - 4}$$

Part II: Rational Exponents – Write final answers in radical form.

$$15. 2^{\frac{1}{2}} \cdot 2^3 = 2^{7/2} = \sqrt{2^7} = \sqrt{2^6} \sqrt{2} = 2^3 \sqrt{2}$$

$$16. \frac{p^{\frac{5}{3}}}{p^{\frac{1}{3}}} = p^{4/3} = \sqrt[3]{p^4}$$

$$15. 8\sqrt{2}$$

$$16. p^3 \sqrt{p}$$

$$17. (y^2)^{\frac{3}{4}} = y^{6/4} = y^{3/2} = \sqrt{y^3}$$

$$18. \left(\frac{75}{3}\right)^{\frac{2}{3}} = 25^{2/3} = \sqrt[3]{625} = \sqrt[3]{125} \sqrt[3]{5}$$

$$17. y\sqrt{y}$$

$$18. 5\sqrt[3]{5}$$

$$19. \frac{1}{4}$$

$$19. 64^{\frac{2}{3}} \cdot 8^{-1/3} = 16$$

$$20. \frac{\sqrt{c^3}}{\sqrt[4]{c^5}} = \frac{c^{3/2}}{c^{5/4}} = c^{1/4}$$

$$20. \sqrt[4]{c}$$

$$B. 4x^5 y^{12} \sqrt{7xy}$$

BONUS. $\sqrt[4]{16x^2 y^6} \cdot \sqrt{28x^{10} y^{22}}$

$$(16x^2 y^6)^{1/4} \cdot (28x^{10} y^{22})^{1/2} = 2x^{1/2} y^{3/2} \cdot 2\sqrt{7} x^5 y^{11} = 2\sqrt{7} x^{5.5} y^{11.5}$$

Name:	Date:
Topic:	Class:

Main Ideas/Questions	Notes
Solving RADICAL EQUATIONS	① ISOLATE the radical on one side of the equation.
	② RAISE EACH SIDE OF THE EQUATION TO THE POWER OF the index of the radical to eliminate the radical sign.
	③ SOLVE the remaining equation.
	④ CHECK for extraneous solutions.

Directions: Solve each equation. Be sure to check for extraneous solutions.

1. $\sqrt{x} + 5 = 12$ $\begin{array}{r} -5 \quad -5 \\ (\sqrt{x})^2 = (7)^2 \\ \boxed{x = 49} \end{array}$ ck: $\sqrt{49} + 5 = 12$ $7 + 5 = 12 \checkmark$	2. $3 - \sqrt[4]{m} = 0$ $\begin{array}{r} + \sqrt[4]{m} \quad + \sqrt[4]{m} \\ 3 = \sqrt[4]{m} \\ 3^4 = (\sqrt[4]{m})^4 \\ 81 = m \\ \boxed{m = 81} \end{array}$ ck: $3 - \sqrt[4]{81} = 0$ $3 - 3 = 0 \checkmark$
3. $10 + \sqrt{a+7} = 11$ $\begin{array}{r} -10 \quad -10 \\ (\sqrt{a+7})^2 = (1)^2 \\ a+7 = 1 \\ -7 \quad -7 \\ \boxed{a = -6} \end{array}$ ck: $10 + \sqrt{-6+7} = 11$ $10 + 1 = 11 \checkmark$	4. $6w - 5 + \sqrt{w-5} = 3$ $\begin{array}{r} -10 \quad -10 \\ (\sqrt{w-5})^2 = (-7)^2 \\ 6w-5 = 49 \\ +5 \quad +5 \\ 6w = 54 \\ w = 9 \end{array}$ ck: $\sqrt{6(9)-5} + 10 = 3$ $\sqrt{54-5} + 10 = 3$ $\sqrt{49} + 10 = 3$ $7 + 10 \neq 3$ No Solution
5. $-36 = -9(2k+12)^{1/2}$ $\begin{array}{r} -9 \quad -9 \\ (-2)^2 = (-2)^2 \\ 4 = 4 \end{array}$ ck: $-36 = -9(2(0.5)+12)^{1/2}$ $-36 = -9(16)^{1/2}$ $-36 = -9(4) \checkmark$	6. $(7p-1)^{1/3} + 11 = 7$ $\begin{array}{r} -11 \quad -11 \\ ((7p-1)^{1/3})^3 = (-4)^3 \\ 7p-1 = -64 \\ +1 \quad +1 \\ 7p = -63 \\ p = -9 \end{array}$ ck: $(7(-9)-1)^{1/3} + 11 = 7$ $(-63-1)^{1/3} + 11 = 7$ $(-64)^{1/3} + 11 = 7$ $-4 + 11 = 7 \checkmark$
7. $8k+12 = 16$ $\begin{array}{r} -12 \quad -12 \\ 8k = 4 \\ k = 0.5 \end{array}$ ck: $\sqrt[4]{8(0.5)+12} - 9 = -11$ $\sqrt[4]{4+12} - 9 = -11$ $\sqrt[4]{16} - 9 = -11$ $2 - 9 \neq -11$ No Solution	8. $-4\sqrt{43-3x} + 18 = -2$ $\begin{array}{r} -4 \quad -4 \\ (\sqrt{43-3x})^2 = (5)^2 \\ 43-3x = 25 \\ -3x = -18 \\ \boxed{x = 6} \end{array}$ ck: $-4\sqrt{43-3(6)} + 18 = -2$ $-4\sqrt{43-18} + 18 = -2$ $-4\sqrt{25} + 18 = -2$ $-4(5) + 18 = -2$ $-20 + 18 = -2 \checkmark$

9. $\sqrt{u-8} = \sqrt{43-2u}$ $(\sqrt{u-8})^2 = (\sqrt{43-2u})^2$ $u-8 = 43-2u$ $3u-8 = 43$ $3u = 51$ $u = 17$ ck: $\sqrt{17-8} = \sqrt{43-2(17)}$ $\sqrt{9} = \sqrt{43-34}$ $3 = 3 \checkmark$	10. $\sqrt[4]{27-2d} = \sqrt[4]{d-3}$ $(\sqrt[4]{27-2d})^4 = (\sqrt[4]{d-3})^4$ $27-2d = d-3$ $27 = 3d-3$ $30 = 3d$ $10 = d$ ck: $\sqrt[4]{27-2(10)} = \sqrt[4]{10-3}$ $\sqrt[4]{27-20} = \sqrt[4]{7}$ $\sqrt[4]{7} = \sqrt[4]{7} \checkmark$
11. $\frac{4\sqrt{3q+15}}{4} = \frac{12\sqrt{q}}{4}$ $(\sqrt{3q+15})^2 = (\sqrt{3q})^2$ $3q+15 = 9q$ $-3q \quad -3q$ $15 = 6q$ $\frac{5}{2} = q$ ck: $4\sqrt{3(\frac{5}{2})+15} = 12\sqrt{\frac{5}{2}}$ $4\sqrt{\frac{15}{2}+15} = 12\sqrt{\frac{5}{2}}$ $4\sqrt{\frac{45}{2}} = 12\sqrt{\frac{5}{2}}$ $4\sqrt{\frac{9 \cdot 5}{2}} = 12\sqrt{\frac{5}{2}}$ $12\sqrt{\frac{5}{2}} = 12\sqrt{\frac{5}{2}} \checkmark$	12. $(\sqrt[3]{4-10n})^3 = (-2\sqrt[3]{n})^3$ $4-10n = -2^3n$ $4 = -8n$ $-1 = -2n$ $1 = 2n$ $\frac{1}{2} = n$ ck: $\sqrt[3]{4-10(\frac{1}{2})} = -2\sqrt[3]{\frac{1}{2}}$ $\sqrt[3]{4-5} = -2\sqrt[3]{\frac{1}{2}}$ $\sqrt[3]{-1} = -2\sqrt[3]{\frac{1}{2}}$ $-1 = -2\sqrt[3]{\frac{1}{2}} \checkmark$
13. $(\sqrt{8x})^2 = (x)^2$ $8x = x^2$ $0 = x^2 - 8x$ $0 = x(x-8)$ $x=0 \quad x=8$ ck: $\sqrt{8 \cdot 0} = \sqrt{0}$ $0 = 0 \checkmark$	14. $\sqrt{12-a} = 8$ $(\sqrt{12-a})^2 = 8^2$ $12-a = 64$ $-a = 52$ $a = -52$ ck: $\sqrt{12-(-52)} = 8$ $\sqrt{12+52} = 8$ $\sqrt{64} = 8$ $8 = 8 \checkmark$
15. $(\sqrt{6m-38})^2 = (m-5)^2$ $6m-38 = m^2-10m+25$ $0 = m^2-16m+63$ $0 = (m-9)(m-7)$ $m=9 \quad m=7$ ck: $\sqrt{6(9)-38} = 9-5$ $\sqrt{42-38} = 4$ $\sqrt{4} = 4 \checkmark$	16. $(\sqrt{2v+3})^2 = (v+2)^2$ $2v+3 = v^2+4v+4$ $0 = v^2+2v+1$ $0 = (v+1)(v+1)$ $v=-1 \quad v=-1$ ck: $\sqrt{2(-1)+3} = -1+2$ $\sqrt{-2+3} = 1$ $\sqrt{1} = 1 \checkmark$
17. $(\sqrt{y-11})^2 = (y-11)^2$ $y-11 = y^2-22y+121$ $0 = y^2-23y+132$ $0 = (y-10)(y-13)$ $y=10 \quad y=13$ ck: $\sqrt{51-5(10)} = 10-11$ $\sqrt{51-50} = -1$ $\sqrt{1} \neq -1$ $\sqrt{51-5(13)} = 13-11$ $\sqrt{51-65} = -4$ $\sqrt{16} \neq -4$ No Solution	18. $(\sqrt{x+9})^2 = (\sqrt{x-1})^2$ $x+9 = x-2\sqrt{x}+1$ $9 = -2\sqrt{x}+1$ $8 = -2\sqrt{x}$ $(-4)^2 = (\sqrt{x})^2$ $16 = x$ ck: $\sqrt{16+9} = \sqrt{16-1}$ $\sqrt{25} = 4-1$ $5 \neq 3$ No Solution

Name: _____

Unit 6: Radical Functions

Date: _____ Per: _____

Homework 6: Radical Equations

**** This is a 2-page document! ******Directions:** Solve each equation. Be sure to check for extraneous solutions.

1. $\sqrt{3-2v}+17=20$

$$(\sqrt{3-2v})^2 = (3)^2$$

$$3-2v=9$$

$$-2v=6$$

$$\boxed{v=-3}$$

ck:

$$\sqrt{3-2(-3)}+17=20$$

$$\sqrt{9}+17=20$$

$$3+17=20 \checkmark$$

2. $\sqrt{4k-11}+15=2$

$$(\sqrt{4k-11})^2 = (-13)^2$$

$$4k-11=169$$

$$4k=180$$

$$k=45$$

$$\sqrt{180-11}+15=2$$

$$\sqrt{169}+15=2$$

$$13+15=2$$

$$28 \neq 2$$

 $\boxed{\text{No Solution}}$

3. $-7\sqrt[4]{m+1}=14$

$$(\sqrt[4]{m+1})^4 = (-2)^4$$

$$m+1=16$$

$$m \neq 15$$

ck:

$$-7\sqrt[4]{15+1}=14$$

$$-7\sqrt[4]{16}=14$$

$$-7(2) \neq 14$$

 $\boxed{\text{No Solution}}$

4. $(x+7)^{\frac{1}{2}}-1=5$

$$((x+7)^{\frac{1}{2}})^2 = (6)^2$$

$$36 = x+7$$

$$\boxed{29}$$

$$5 = (29+7)^{\frac{1}{2}}-1$$

$$-5 = (36)^{\frac{1}{2}}-1$$

$$-5 = 6-1 \checkmark$$

5. $-7+(16p+8)^{\frac{1}{3}}=-1$

$$((16p+8)^{\frac{1}{3}})^3 = (6)^3$$

$$16p+8=216$$

$$16p=208$$

$$p=13$$

$$(16(13)+8)^{\frac{1}{3}}=-1$$

$$216^{\frac{1}{3}}=-1$$

$$-7+6=-1 \checkmark$$

6. $17-2\sqrt{3x+10}=1$

$$-2\sqrt{3x+10}=-16$$

$$(\sqrt{3x+10})^2 = (8)^2$$

$$3x+10=64$$

$$3x=54$$

$$\boxed{x=18}$$

ck:

$$17-2\sqrt{3(18)+10}=1$$

$$17-2\sqrt{64}=1$$

$$17-16=1 \checkmark$$

7. $(\sqrt{3k-29})^2 = (\sqrt{41-2k})^2$

$$29=41-2k$$

$$5k=70$$

$$\boxed{k=14}$$

ck:

$$\sqrt{3(14)-29} = \sqrt{41-2(14)}$$

$$\sqrt{42-29} = \sqrt{41-28}$$

$$\sqrt{13} = \sqrt{13} \checkmark$$

8. $((3x-21)^{\frac{1}{3}})^3 = ((19-x)^{\frac{1}{3}})^3$

$$3x-21=19-x$$

$$4x-21=19$$

$$4x=40$$

$$\boxed{x=10}$$

ck:

$$(3(10)-21)^{\frac{1}{3}} = (19-10)^{\frac{1}{3}}$$

$$(30-21)^{\frac{1}{3}} = 9^{\frac{1}{3}}$$

$$9^{\frac{1}{3}} = 9^{\frac{1}{3}} \checkmark$$

$$9. (\sqrt{19x+24})^2 = (5\sqrt{x})^2$$

$$19x+24 = 25x$$

$$24 = 6x$$

$$x \neq 4$$

ck:

$$\sqrt{19 \cdot 4 + 24} = -5\sqrt{4}$$

$$\sqrt{100} \neq -10$$

No Solution

$$10. \frac{\sqrt[4]{7w-5}}{2} = \sqrt[4]{w-2}$$

$$(\sqrt[4]{7w-5})^4 = (2\sqrt[4]{w-2})^4$$

$$7w-5 = 16(w-2)$$

$$7w-5 = 16w-32$$

$$27 = 9w$$

$$w = 3$$

ck:

$$\sqrt[4]{7 \cdot 3 - 5}$$

$$\sqrt[4]{3-2}$$

$$2$$

$$\sqrt[4]{16}$$

$$2$$

$$11. (\sqrt{63-2x})^2 = x^2$$

$$63-2x = x^2$$

$$0 = x^2 + 2x - 63$$

$$0 = (x+9)(x-7)$$

$$x = -9 \quad x = 7$$

ck:

$$\sqrt{63 - (-18)} = -9$$

$$\sqrt{81} \neq -9$$

$$\sqrt{63 - 14} = 7$$

$$\sqrt{49} = 7$$

$$12. n = (12 -$$

$$n^2 =$$

$$-4 = (12+4)^{1/2}$$

$$4 \neq 16^{1/2}$$

$$3 = (12-3)^{1/2}$$

$$3 = 9^{1/2} \checkmark$$

$$13. \sqrt{5y-15} = y-3$$

$$5y-15 = y^2 - 6y$$

$$0 = y^2 - 11y + 15$$

$$0 = (y-3)(y-5)$$

$$y = 3 \quad y = 5$$

ck:

$$\sqrt{40-15} = 8-3$$

$$\sqrt{25} = 5 \checkmark$$

$$\sqrt{15-15} = 5-3$$

$$\sqrt{0} = 2 \checkmark$$

$$6c+15 = c+4$$

$$5c+15 = c^2 + 8c + 16$$

$$0 = c^2 + 3c + 1$$

$$0 = (c+1)(c+1)$$

$$c = -1 \quad c = -1$$

ck:

$$\sqrt{-6+15} = -1+4$$

$$\sqrt{9} = 3 \checkmark$$

$$15. \sqrt{4a+7} = a+7$$

$$\sqrt{4a+7} = a+7$$

$$4a+7 = a^2 + 14a + 49$$

$$0 = a^2 + 10a + 42$$

$$0 = (a+4)(a+7)$$

$$a = -4 \quad a = -7$$

ck:

$$\sqrt{41-32} + 7 = 4$$

$$\sqrt{9} + 7 = 4$$

$$10 \neq 4$$

$$\sqrt{41-16} + 7 = 2$$

$$\sqrt{25} + 7 = 2$$

$$12 \neq 2$$

No Solution

$$16. \sqrt{q-21} = \sqrt{q} + 7$$

$$q-21 = q + 14\sqrt{q} + 49$$

$$-21 = 14\sqrt{q} + 49$$

$$-70 = 14\sqrt{q}$$

$$-5 = \sqrt{q}$$

$$25 \neq q$$

ck:

$$\sqrt{25-21} = \sqrt{25} + 7$$

$$\sqrt{4} = 5 + 7$$

$$2 \neq 12$$

No Solution

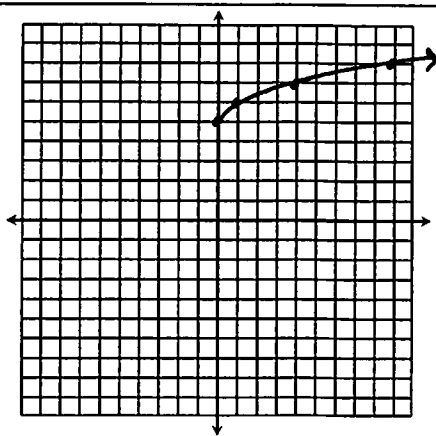
Name:	Date:
Topic:	Class:

Main Ideas/Questions	Notes/Examples	
SQUARE ROOT <i>Function</i> Parent Function: $f(x) = \sqrt{x}$	A radical function is a function of the form $f(x) = \sqrt[n]{x}$. The square root function is a type of radical function. Graph the parent function of the square root function below and identify the key characteristics.	
		D: $\{x \mid x \geq 0\}$ y-intercept: $(0, 0)$ As $x \rightarrow \infty$, $f(x) \rightarrow \infty$ As $x \rightarrow 0$, $f(x) \rightarrow 0$ Increasing interval(s): $[0, \infty)$ Decreasing interval(s): N/A
TRANSFORMATIONS $f(x) = a\sqrt{x-h} + k$	h is the <u>horizontal</u> shift. (+ shifts <u>left</u>, - shifts <u>right</u>) k is the <u>vertical</u> shift. (+ shifts <u>up</u>, - shifts <u>down</u>) (h, k) is the <u>endpoint</u> - If $a < 0$, the function is <u>reflected</u> across the <u>x-axis</u> $a > 1$ represents a vertical <u>stretch</u> $0 < a < 1$ represents a vertical <u>compression</u>	
	Identify the transformations of each function compared to the parent function.	
	1. $f(x) = \sqrt{x-7} + 2$ Right 7, up 2	2. $f(x) = 4\sqrt{x} - 1$ Vert. stretch by 4, down 1
	3. $f(x) = -\sqrt{x+3}$ Reflect across x-axis, left 3	4. $f(x) = \frac{1}{2}\sqrt{x-2} - 6$ Vert. compression by $\frac{1}{2}$, right 2, down 6
	5. The square root parent function is reflected across the x-axis, then shifted 7 units left and 1 unit down. Write an equation that represents this function. $f(x) = -\sqrt{x+7} - 1$	

GRAPHING SQUARE ROOT FUNCTIONS

Graph each function. Identify the key characteristics.

6. $f(x) = \sqrt{x} + 5$



D: $\{x | x \geq 0\}$ R: $\{y | y \geq 5\}$

Endpoint: $(0, 5)$

End Behavior:

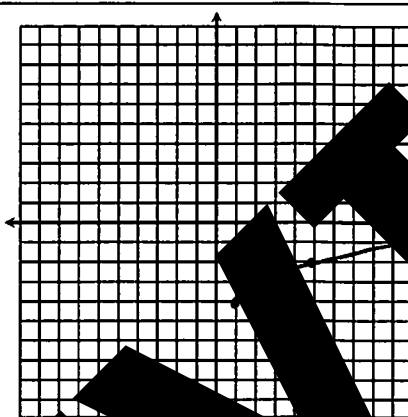
As $x \rightarrow \infty$, $f(x) \rightarrow \infty$

As $x \rightarrow 0$, $f(x) \rightarrow 5$

Increasing Interval(s): $[0, \infty)$

Decreasing Interval(s): N/A

7. $f(x) = \sqrt{x-1} - 4$



D: $\{x | x \geq 1\}$ R: $\{y | y \geq -4\}$

Endpoint: $(1, -4)$

End Behavior:

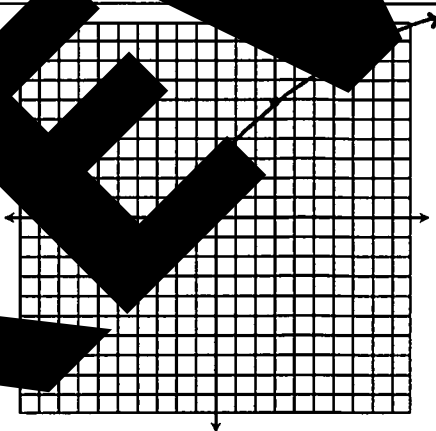
As $x \rightarrow \infty$, $f(x) \rightarrow \infty$

As $x \rightarrow 1$, $f(x) \rightarrow -4$

Increasing Interval(s): $[1, \infty)$

Decreasing Interval(s): N/A

8. $f(x) = 3\sqrt{x+1}$



D: $\{x | x \geq -1\}$ R: $\{y | y \geq 0\}$

Endpoint: $(-1, 0)$

End Behavior:

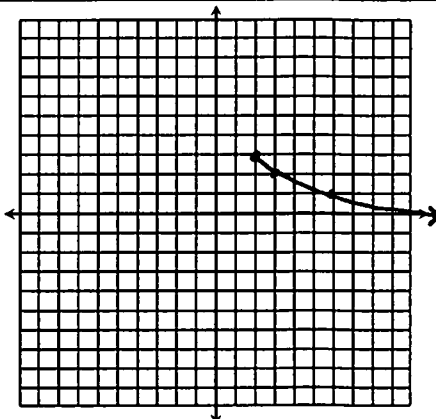
As $x \rightarrow \infty$, $f(x) \rightarrow \infty$

As $x \rightarrow -1$, $f(x) \rightarrow 0$

Increasing Interval(s): $[-1, \infty)$

Decreasing Interval(s): N/A

9. $f(x) = -\sqrt{x-2} + 3$



D: $\{x | x \geq 2\}$ R: $\{y | y \leq 3\}$

Endpoint: $(2, 3)$

End Behavior:

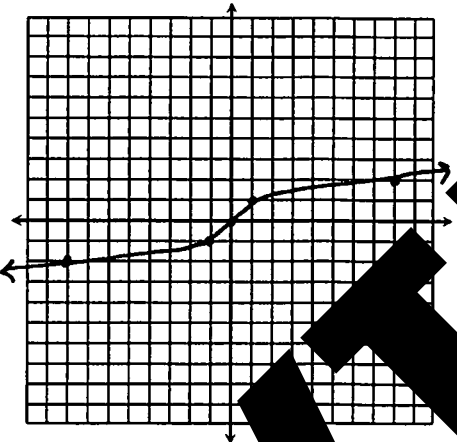
As $x \rightarrow \infty$, $f(x) \rightarrow -\infty$

As $x \rightarrow 2$, $f(x) \rightarrow 3$

Increasing Interval(s): N/A

Decreasing Interval(s): $[2, \infty)$

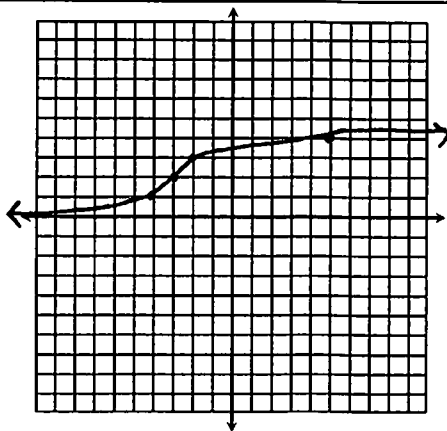
Name:	Date:
Topic:	Class:

Main Ideas/Questions	Notes/Examples
CUBE ROOT Function Parent Function: $f(x) = \sqrt[3]{x}$	The cube root function is another type of radical function. Graph the parent function of the cube root below and identify the key characteristics.  D: <u>TP</u> R: <u>2</u> Point of Inflection: <u>(0, 0)</u> Behavior: <u>increasing</u> $f(x) \rightarrow \infty$ as $x \rightarrow \infty$ $f(x) \rightarrow -\infty$ as $x \rightarrow -\infty$ Increasing interval(s): <u>$(-\infty, \infty)$</u> Decreasing interval(s): <u>N/A</u>
TRANSFORMATIONS $f(x) = a\sqrt[3]{x-h} + k$	h is the <u>horizontal</u> shift. (+ shifts <u>right</u>, - shifts <u>left</u>) k is the <u>vertical</u> shift. (+ shifts <u>up</u>, - shifts <u>down</u>) - Point of Inflection: <u>(h, k)</u> - If a is negative, the function is <u>reflected</u> across the <u>X-axis</u>. $a > 1$ represents a vertical <u>stretch</u>. $0 < a < 1$ represents a vertical <u>compression</u>. Describe transformations of each function compared to the parent function. 1. $f(x) = \sqrt[3]{x} + 9$ up 9 2. $f(x) = 5\sqrt[3]{x-2}$ Vert. stretch by 5, right 2 3. $f(x) = -\sqrt[3]{x+4} - 7$ Reflect across x-axis, Left 4, down 7 4. $f(x) = \frac{3}{4}\sqrt[3]{x-6} + 1$ Vert. compression by $\frac{3}{4}$, Right 6, up 1 5. The cube root parent function is vertically stretched by a factor of 3, then translated so that its point of inflection is located at (-2, 3). Write an equation that represents this new function. $f(x) = 3\sqrt[3]{x+2} + 3$

GRAPHING CUBE ROOT FUNCTIONS

Graph each function. Identify the key characteristics.

6. $f(x) = \sqrt[3]{x+3} + 2$



D: $\mathbb{R}$ R: $\mathbb{R}$

Point of Inflection: $(-3, 2)$

End Behavior:

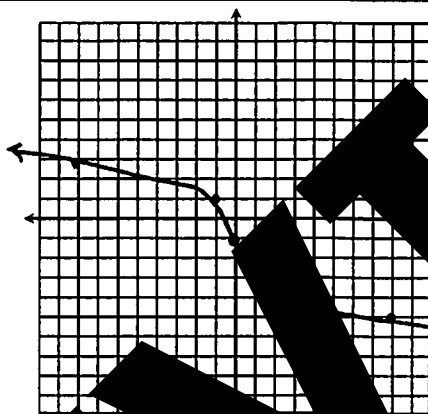
As $x \rightarrow \infty$, $f(x) \rightarrow \infty$

As $x \rightarrow -\infty$, $f(x) \rightarrow -\infty$

Increasing Interval(s): $(-\infty, \infty)$

Decreasing Interval(s): None

7. $f(x) = -2\sqrt[3]{x} - 1$



D: $\mathbb{R}$ R: $\mathbb{R}$

Point of Inflection: $(0, -1)$

End Behavior:

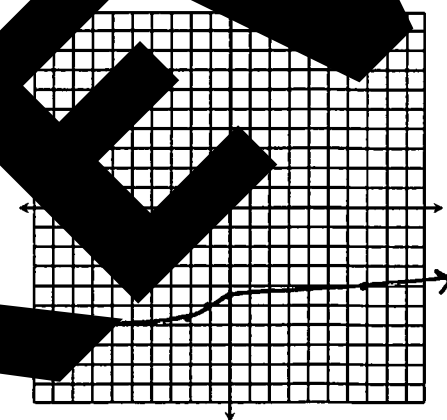
As $x \rightarrow \infty$, $f(x) \rightarrow -\infty$

As $x \rightarrow -\infty$, $f(x) \rightarrow \infty$

Increasing Interval(s): None

Decreasing Interval(s): $(-\infty, \infty)$

8. $f(x) = \frac{1}{2}\sqrt[3]{x+1} - 5$



D: $\mathbb{R}$ R: $\mathbb{R}$

Point of Inflection: $(-1, -5)$

End Behavior:

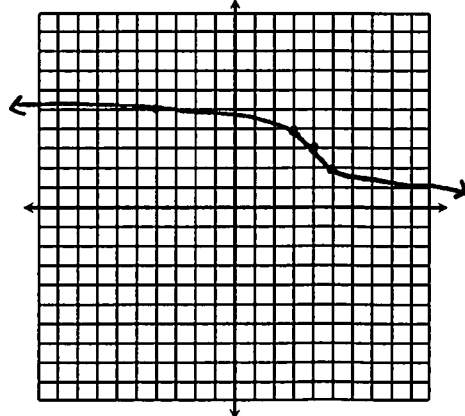
As $x \rightarrow \infty$, $f(x) \rightarrow \infty$

As $x \rightarrow -\infty$, $f(x) \rightarrow -\infty$

Increasing Interval(s): $(-\infty, \infty)$

Decreasing Interval(s): None

9. $f(x) = \sqrt[3]{x-4} + 3$



D: $\mathbb{R}$ R: $\mathbb{R}$

Point of Inflection: $(4, 3)$

End Behavior:

As $x \rightarrow \infty$, $f(x) \rightarrow \infty$

As $x \rightarrow -\infty$, $f(x) \rightarrow -\infty$

Increasing Interval(s): $(-\infty, \infty)$

Decreasing Interval(s): None

SQUARE ROOT Function

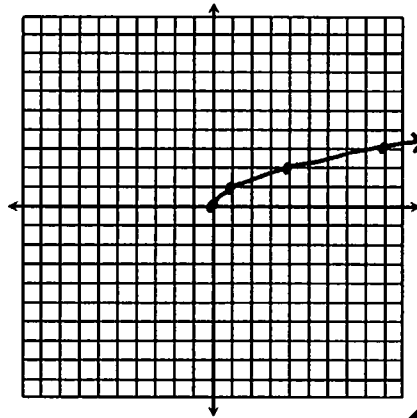
PARENT FUNCTION:

$$f(x) = \sqrt{x}$$

Domain: $\{x | x \geq 0\}$

Range: $\{y | y \geq 0\}$

Endpoint: $(0, 0)$



CUBE ROOT Function

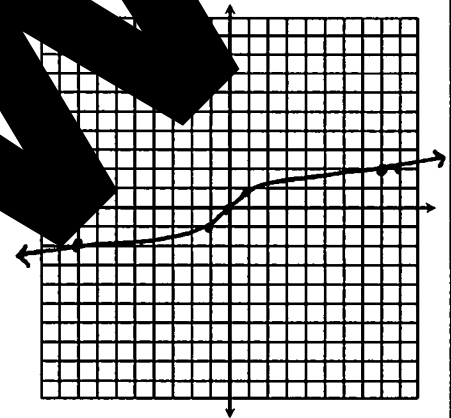
PARENT FUNCTION:

$$f(x) = \sqrt[3]{x}$$

Domain: $\mathbb{R}$

Range: $\mathbb{R}$

Point of Inflection: $(0, 0)$



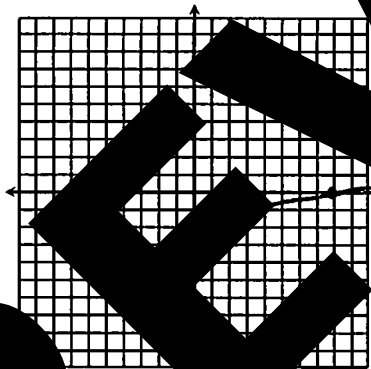
..... Transformations

EXAMPLE 1: $f(x) = \sqrt{x+1} - 3$

Domain: $\{x | x \geq -1\}$

Range: $\{y | y \geq -3\}$

Endpoint: $(-1, -3)$

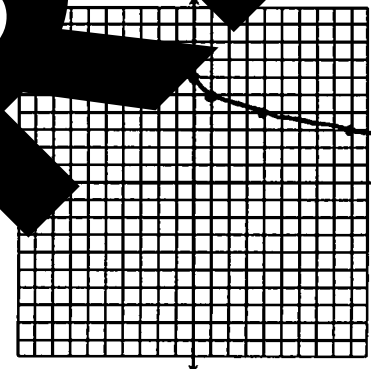


EXAMPLE 2: $f(x) = -\sqrt{x-6}$

Domain: $\{x | x \geq 6\}$

Range: $\{y | y \leq 0\}$

Endpoint: $(6, 0)$

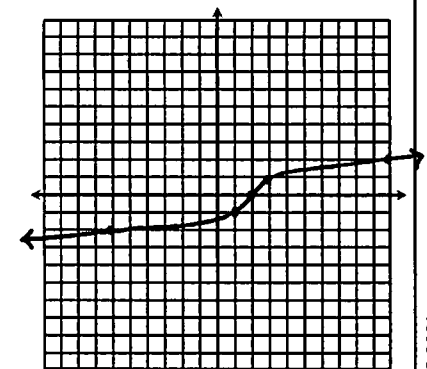


EXAMPLE 1: $f(x) = \sqrt[3]{x} - 2$

Domain: $\mathbb{R}$

Range: $\mathbb{R}$

Point of Inflection: $(2, 0)$

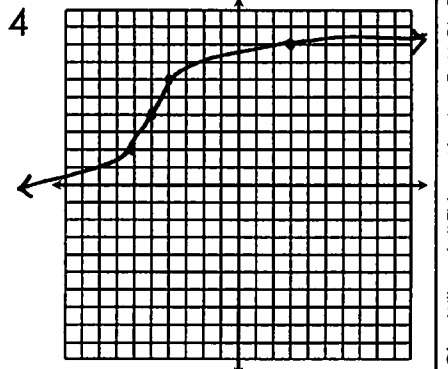


EXAMPLE 2: $f(x) = 2\sqrt[3]{x+5} + 4$

Domain: $\mathbb{R}$

Range: $\mathbb{R}$

Point of Inflection: $(-5, 4)$



Name: _____

Unit 6: Radical Functions

Date: _____ Per: _____

Homework 7: Graphing Radical Functions

**** This is a 2-page document! ******For questions 1-2:** Describe the transformations from the parent function.

1. $f(x) = 2\sqrt{x} - 9$

Vert. stretch by 2,
down 9

2. $f(x) = -\sqrt[3]{x+5} + 3$

Reflect across x-axis,
left 5, up 3

3. $f(x) = \frac{1}{4}\sqrt{x}$

Vert. compression
by $\frac{1}{4}$, right 7

4. The cube root parent function is vertically stretched by a factor of 3, reflected in the x-axis, then translated 8 units down. Write an equation to represent the new function.

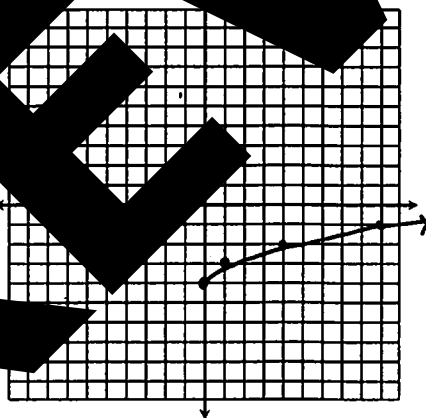
$$f(x) = -3\sqrt[3]{x} - 8$$

5. The square root parent function is vertically compressed by a factor of $\frac{1}{3}$, then translated so that it has an endpoint located at (4, -1). Write an equation to represent the new function.

$$f(x) = \frac{1}{3}\sqrt{x-4} - 1$$

Directions: Graph each function and label its characteristics.

6. $f(x) = \sqrt{x} - 4$



D: $\{x | x \geq 0\}$ R: $\{y | y \geq -4\}$

Endpoint: $(0, -4)$

End Behavior:

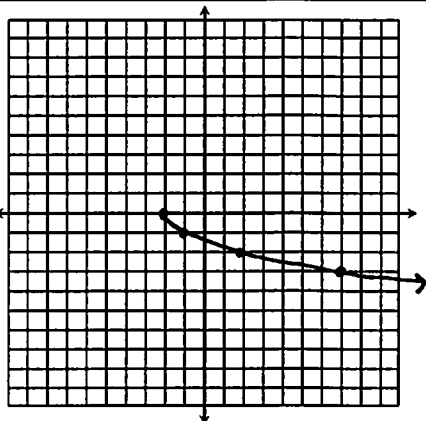
As $x \rightarrow \infty$, $f(x) \rightarrow \infty$

As $x \rightarrow 0$, $f(x) \rightarrow -4$

Increasing Interval(s): $[0, \infty)$

Decreasing Interval(s): N/A

7. $f(x) = \sqrt{x+2}$



D: $\{x | x \geq -2\}$ R: $\{y | y \geq 0\}$

Endpoint: $(-2, 0)$

End Behavior:

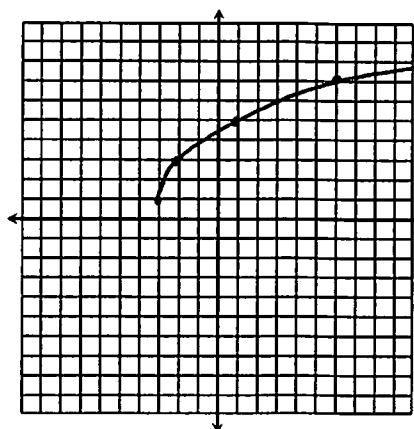
As $x \rightarrow \infty$, $f(x) \rightarrow \infty$

As $x \rightarrow -2$, $f(x) \rightarrow 0$

Increasing Interval(s): $[-2, \infty)$

Decreasing Interval(s): N/A

8. $f(x) = 2\sqrt{x+3} + 1$



D: $\{x | x \geq -3\}$ R: $\{y | y \geq 1\}$

Endpoint: $(-3, 1)$

End Behavior:

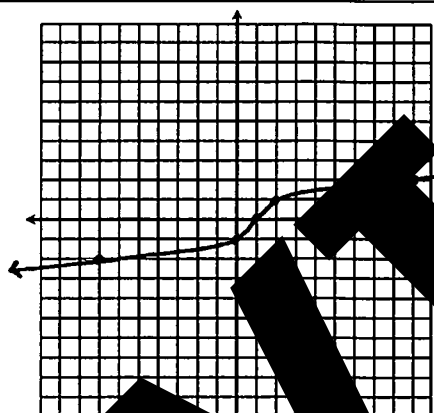
As $x \rightarrow \infty$, $f(x) \rightarrow \infty$

As $x \rightarrow -3$, $f(x) \rightarrow 1$

Increasing Interval(s): $[-3, \infty)$

Decreasing Interval(s): N/A

9. $f(x) = \sqrt[3]{x-1}$



D: $\mathbb{R}$ R: $\mathbb{R}$

Point of Inflection: $(1, 0)$

End Behavior:

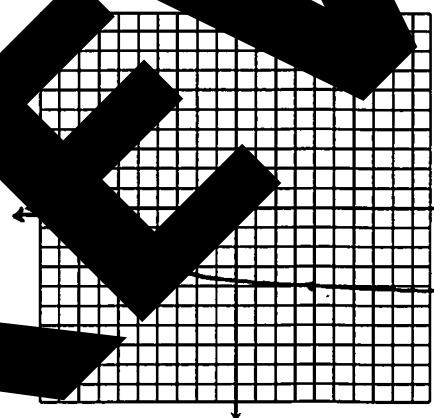
As $x \rightarrow \infty$, $f(x) \rightarrow \infty$

As $x \rightarrow -\infty$, $f(x) \rightarrow -\infty$

Increasing Interval(s): $(-\infty, \infty)$

Decreasing Interval(s): N/A

10. $f(x) = -\sqrt[3]{x+4} - 2$



D: $\mathbb{R}$ R: $\mathbb{R}$

Point of Inflection: $(-4, -2)$

End Behavior:

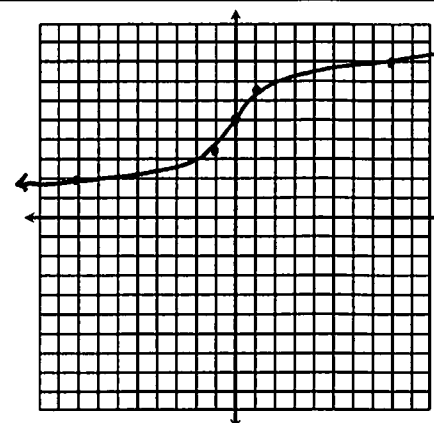
As $x \rightarrow \infty$, $f(x) \rightarrow -\infty$

As $x \rightarrow -\infty$, $f(x) \rightarrow \infty$

Increasing Interval(s): N/A

Decreasing Interval(s): $(-\infty, \infty)$

11. $f(x) = \sqrt[3]{x} + 5$



D: $\mathbb{R}$ R: $\mathbb{R}$

Point of Inflection: $(0, 5)$

End Behavior:

As $x \rightarrow \infty$, $f(x) \rightarrow \infty$

As $x \rightarrow -\infty$, $f(x) \rightarrow -\infty$

Increasing Interval(s): $(-\infty, \infty)$

Decreasing Interval(s): N/A

Name: _____

Algebra 2

Date: _____ Per: _____

Unit 6: Radical Functions

Quiz 6-3: Radical Equations & Graphing Radical Functions

Part I: Radical Equations – Solve each equation. Be sure to check for extraneous solutions.

1. $-2\sqrt{6x+3} = -18$

$$\sqrt{6x+3} = 9$$

$$6x+3 = 81$$

$$6x = 78$$

$$x = 13$$

ck:

$$-2\sqrt{6 \cdot 13 + 3} = -18$$

$$-2\sqrt{81} = -18$$

$$-2 \cdot 9 = -18 \checkmark$$

2. $\sqrt{m-7} + 18 = 12$

$$\sqrt{m-7} = -6$$

$$m-7 = 36$$

$$m = 43$$

ck:

$$\sqrt{43-7} + 18 = 12$$

$$+18 = 12$$

$$+18 = 12$$

3. $(41k-31)^{\frac{1}{4}} = 5$

$$41k-31 = 625$$

$$41k = 656$$

$$k = 16$$

ck:

$$(41 \cdot 16 - 31)^{\frac{1}{4}} = 5$$

$$(656 - 31)^{\frac{1}{4}} = 5$$

$$(625)^{\frac{1}{4}} = 5 \checkmark$$

4. $(45-21)^2 = (a-6)^2$

$$a = a-6$$

$$= 3a$$

$$(45-21)^{\frac{1}{2}} = (17-6)^{\frac{1}{2}}$$

$$(45-34)^{\frac{1}{2}} = 11^{\frac{1}{2}}$$

$$11^{\frac{1}{2}} = 11^{\frac{1}{2}} \checkmark$$

5. $\sqrt[3]{19y+40} = 3\sqrt[3]{y}$

$$19y+40 = 27y$$

$$40 = 8y$$

$$5 = y$$

$$\sqrt[3]{35} = 3\sqrt[3]{5} \checkmark$$

6. $\sqrt{72-n} = n$

$$72-n = n^2$$

$$0 = n^2 + n - 72$$

$$0 = (n+9)(n-8)$$

$$n = -9 \quad n = 8$$

8. $p = \sqrt{4p+8} - 3$

$$p+3 = \sqrt{4p+8}$$

$$p^2 + 6p + 9 = 4p + 8$$

$$p^2 + 2p + 1 = 0$$

$$(p+1)(p+1) = 0$$

$$p = -1 \quad p = -1$$

ck:

$$-1 = \sqrt{4(-1)+8} - 3$$

$$-1 = \sqrt{4-8} - 3$$

$$-1 = 2 - 3 \checkmark$$

ck:

$$(3 \cdot 10 - 21)^{\frac{1}{2}} = 10 - 7$$

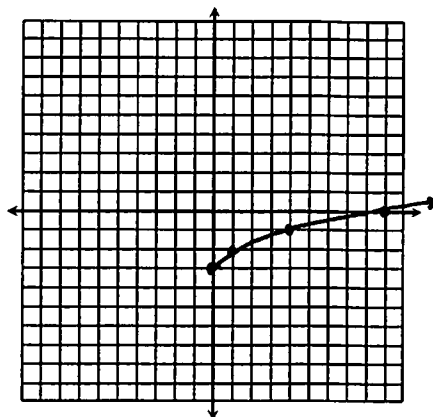
$$9^{\frac{1}{2}} = 3 \checkmark$$

$$(3 \cdot 7 - 21)^{\frac{1}{2}} = 7 - 7$$

$$0^{\frac{1}{2}} = 0 \checkmark$$

Part II: Graphing – Graph each function and identify its key characteristics.

9. $f(x) = \sqrt{x} - 3$



D: $\{x | x \geq 0\}$ R: $\{y | y \geq -3\}$

Endpoint: $(0, -3)$

End Behavior:

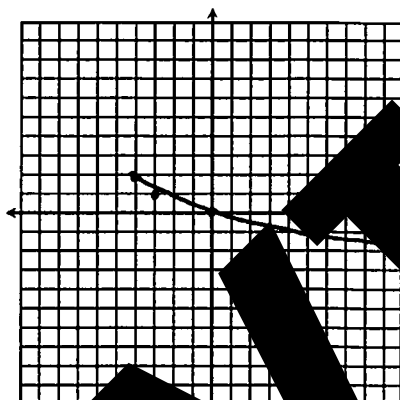
As $x \rightarrow \infty$, $f(x) \rightarrow \infty$

As $x \rightarrow 0$, $f(x) \rightarrow -3$

Increasing Interval(s):

Decreasing Interval(s): N/A

10. $f(x) = -\sqrt{x+4} + 2$



D: $\{x | x \geq -4\}$ R: $\{y | y \leq 2\}$

Endpoint: $(-4, 2)$

End Behavior:

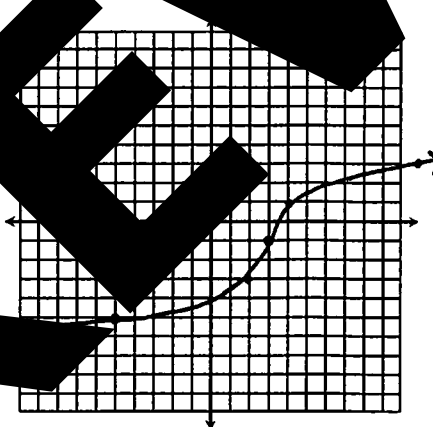
As $x \rightarrow \infty$, $f(x) \rightarrow -\infty$

As $x \rightarrow -4$, $f(x) \rightarrow 2$

Increasing Interval(s): N/A

Decreasing Interval(s): $[-4, \infty)$

11. $f(x) = 2\sqrt[3]{x-3} - 1$



D: $\mathbb{R}$ R: $\mathbb{R}$

Point of Inflection: $(3, -1)$

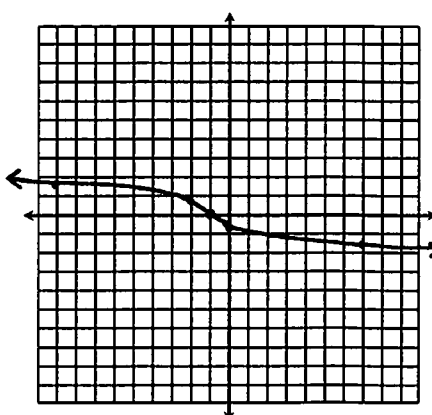
End Behavior:

As $x \rightarrow \infty$, $f(x) \rightarrow \infty$

As $x \rightarrow -\infty$, $f(x) \rightarrow -\infty$

Increasing Interval(s): $(-\infty, \infty)$

Decreasing Interval(s): N/A



D: $\mathbb{R}$ R: $\mathbb{R}$

Point of Inflection: $(-1, 0)$

End Behavior:

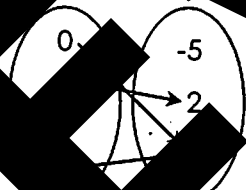
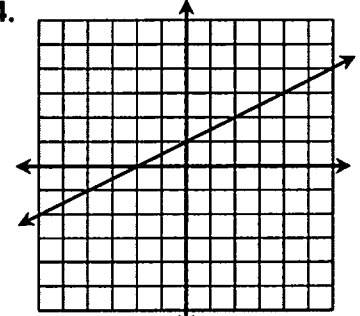
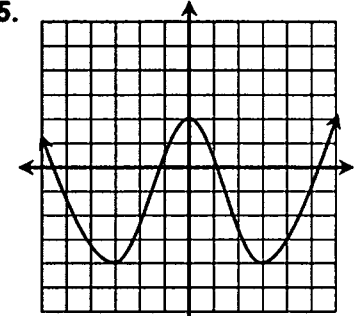
As $x \rightarrow \infty$, $f(x) \rightarrow -\infty$

As $x \rightarrow -\infty$, $f(x) \rightarrow \infty$

Increasing Interval(s): N/A

Decreasing Interval(s): $(-\infty, \infty)$

Name:	Date:
Topic:	Class:

Main Ideas/Questions	Notes/Examples
INVERSE RELATIONS	Notation: The inverse of a relation R is written as R^{-1} To find the inverse of a relation: <u>Switch the x values</u>
ONE-TO-ONE Relationship	A function has a one-to-one relationship if the relation <u>and its inverse</u> are functions.
EXAMPLES	1. $R: \{(1, -2), (2, 4), (3, -1), (4, -2)\}$ $R^{-1}: \{(-2, 1), (4, 2), (-1, 3), (-2, 4)\}$ Is R a function? <u>Y</u> Is R^{-1} a function? <u>N</u> Is R one-to-one? <u>N</u> 2. $R: \{(1, 5), (1, 6), (2, 3), (4, 9)\}$ $R^{-1}: \{(5, 1), (6, 1), (3, 2), (9, 4)\}$ Is R a function? <u>N</u> Is R^{-1} a function? <u>Y</u> Is R one-to-one? <u>N</u> 3. $R: \{(6, 0), (6, 11), (2, 1)\}$  Is R a function? <u>Y</u> Is R^{-1} a function? <u>N</u> Is R one-to-one? <u>N</u>
HORIZONTAL LINE TEST	• Recall: The vertical line test is used to determine whether a relation is a function given a graph. • The horizontal line test is used to determine whether the <u>inverse</u> of a relation is a function. • BOTH the relation and its inverse must be a function in order to be a one-to-one function.
EXAMPLES Determine whether the relation shown represents a one-to-one function.	4.  <u>Yes</u> 5.  <u>No</u>

INVERSE FUNCTIONS

Notation: The inverse of a function $f(x)$ is written as $f^{-1}(x)$.

To find the inverse of a function:

- **Step 1:** Replace $f(x)$ with y .
- **Step 2:** Interchange x with y .
- **Step 3:** Solve for y .
- **Step 4:** Rewrite y as $f^{-1}(x)$

EXAMPLES

Find the inverse of each function.

6. $f(x) = 5x$
 $y = 5x \rightarrow x = 5y$
 $\frac{x}{5} = y$

$$f^{-1}(x) = \frac{x}{5}$$

7. $f(x) = x - 7$
 $y = x - 7 \rightarrow x = y + 7$

$$f^{-1}(x) = x + 7$$

8. $f(x) = \frac{x-8}{3}$
 $y = \frac{x-8}{3} \rightarrow x = \frac{y-8}{3}$

$$3x = y - 8$$

$$3x + 8 = y$$

$$f^{-1}(x) = x + 8$$

$y = \frac{1}{5}x + 1$
 $x = 5(y - 1)$
 $x = 5y - 5$
 $x + 5 = 5y$
 $\frac{x+5}{5} = y$

$$f^{-1}(x) = \frac{x+5}{5}$$

10. $f(x) = x^2 + 9$ (if $x \geq 0$)
 $y = x^2 + 9$
 $x^2 = y - 9$
 $x = \sqrt{y - 9}$
 $f^{-1}(x) = \sqrt{x - 9}; x \geq 9$

$f(x) = (x+4)^3 - 6$
 $y = (x+4)^3 - 6 \rightarrow x + 4 = \sqrt[3]{y + 6}$
 $x = \sqrt[3]{y + 6} - 4$
 $f^{-1}(x) = \sqrt[3]{x + 6} - 4$

VERIFYING INVERSES

using Composition

Functions, f and g , are inverse functions if and only if:

$$f(g(x)) = x \quad \text{AND} \quad g(f(x)) = x$$

EXAMPLES

Determine whether the functions are inverse functions.

12. $f(x) = x^2 + 8$ (if $x \geq 0$) and $g(x) = \sqrt{x - 8}$

$$f(g(x)) = (\sqrt{x - 8})^2 + 8$$

$$= x - 8 + 8$$

$$= x \quad \checkmark$$

$$g(f(x)) = \sqrt{(x^2 + 8) - 8}$$

$$= \sqrt{x^2}$$

$$= x \quad \checkmark$$

Yes, inverses

13. $f(x) = -4x + 5$ and $g(x) = \frac{x}{4} - 5$

$$f(g(x)) = -4\left(\frac{x}{4} - 5\right) + 5$$

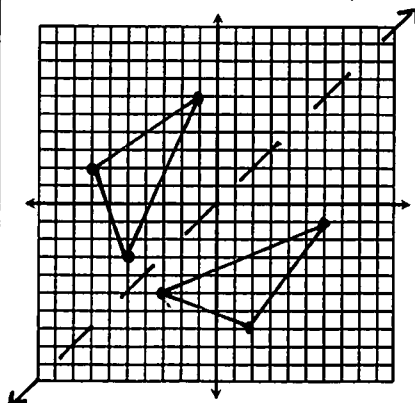
$$= -x + 20 + 5$$

$$= -x + 25$$

No, not inverses

Graphing INVERSE FUNCTIONS

Given: $\triangle ABC$ is formed by the points in the relation $\{(-1, 6), (-7, 2), (-5, -3)\}$



a) What is the inverse of this relation?

$$\{(6, -1), (2, -7), (-3, -5)\}$$

b) Graph $\triangle ABC$ and its inverse. What do you notice about the two triangles?

Reflection across the line $y = x$

Find the inverse of each function. Then, graph both functions and label the inverse relationship on the graph.

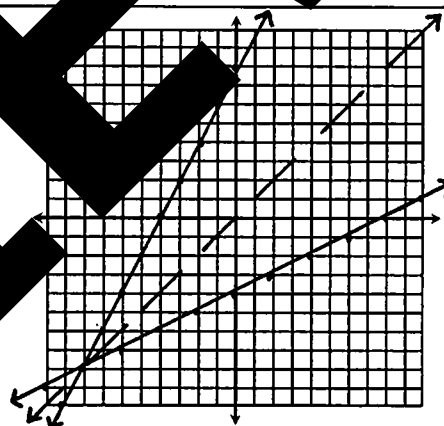
14. $f(x) = 2x + 8$

$$x = 2y + 8$$

$$x - 8 = 2y$$

$$\frac{1}{2}x - 4 = y$$

$$f^{-1}(x) = \frac{1}{2}x - 4$$

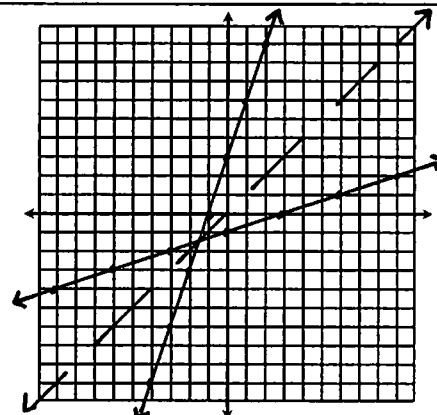


15. $f(x) = \frac{1}{3}x + 1$

$$x = \frac{1}{3}y + 1$$

$$3x + 3 = y$$

$$f^{-1}(x) = 3x + 3$$



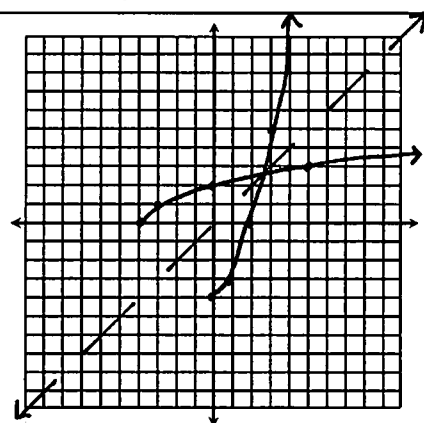
16. $f(x) = x^2 - 4$ (if $x \geq 0$)

$$x = y^2 - 4$$

$$x + 4 = y^2$$

$$\sqrt{x + 4} = y$$

$$f^{-1}(x) = \sqrt{x + 4}; x \geq -4$$



Name: _____

Unit 6: Radical Functions

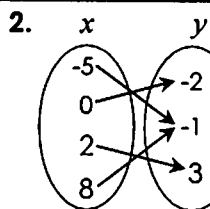
Date: _____ Per: _____

Homework 8: Inverse Relations & Functions

**** This is a 2-page document! ******Give the inverse of each relation, then determine whether it is a one-to-one function.**

1. $R = \{(-2, -7), (-1, -3), (0, 5), (1, 2)\}$

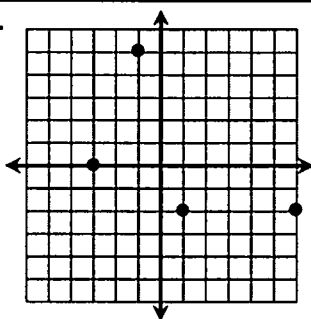
$R^{-1} = \{(-7, -2), (-3, -1), (5, 0), (2, 1)\}$

Yes; one-to-one

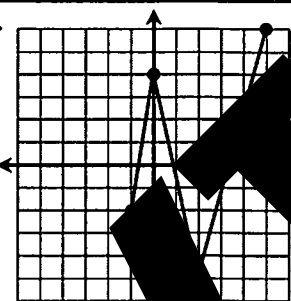
$R^{-1} = \{(-2, -5), (-1, 0), (3, 2), (3, 8)\}$

No; not one-to-one**Determine whether the relation shown on the graph represents a one-to-one function.**

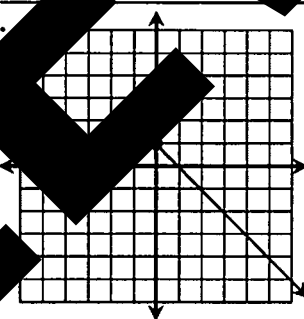
3.

No

4.

No

5.

Yes**Find the inverse of each function.**

6. $f(x) = -x + 9$

$x = -y + 9$

$x - 9 = -y$

$-x + 9 = y$

$f^{-1}(x) = -x + 9$

$f(x) = \frac{4}{3}x - 7$

$x = \frac{4}{3}y - 7$

$x + 7 = \frac{4}{3}y$

$\frac{3}{4}x + \frac{21}{4} = y$

$f^{-1}(x) = \frac{3}{4}x + \frac{21}{4}$

8. $f(x) = (x+5)^2$

$x = (y+5)^2$

$x + 5 = \sqrt{y}$

$(x+5)^2 = y$

$f^{-1}(x) = (x+5)^2$
 $x \geq -5$

9. $f(x) = (x-2)^4$ (if $x \geq 2$)

$x = (y-2)^4$

$\sqrt[4]{x} = y - 2$

$\sqrt[4]{x} + 2 = y$

$f^{-1}(x) = \sqrt[4]{x} + 2$
 $x \geq 0$

Determine whether the pair of functions are inverse functions.

1. $f(x) = 6x - 15$ and $g(x) = \frac{1}{6}x + \frac{5}{2}$

$f(g(x)) = 6\left(\frac{1}{6}x + \frac{5}{2}\right) - 15$

$= x + 15 - 15$

$= x \checkmark$

$g(f(x)) = \frac{1}{6}(6x - 15) + \frac{5}{2}$

$= x - \frac{5}{2} + \frac{5}{2}$

$= x \checkmark$

Yes, inverses

11. $f(x) = \frac{1}{2}x - \frac{7}{2}$ and $g(x) = 2x + 14$

$$\begin{aligned} f(g(x)) &= \frac{1}{2}(2x+14) - \frac{7}{2} \\ &= x + 7 - \frac{7}{2} \\ &= x + \frac{7}{2} \end{aligned}$$

No, not inverses

12. $f(x) = \sqrt{x+4} + 9$ and $g(x) = (x-9)^2 - 4$ (if $x \geq 9$)

$$\begin{aligned} f(g(x)) &= \sqrt{(x-9)^2 - 4 + 4} + 9 \\ &= \sqrt{(x-9)^2} + 9 \\ &= x - 9 + 9 \\ &= x \quad \checkmark \end{aligned}$$

$$\begin{aligned} g(f(x)) &= ((\sqrt{x+4} + 9) - 9)^2 - 4 \\ &= (\sqrt{x+4})^2 - 4 \\ &= x + 4 - 4 \\ &= x \quad \checkmark \end{aligned}$$

Yes, inverses

13. $f(x) = \frac{1}{4}x^3 - 3$ and $g(x) = \sqrt[3]{4x-12}$

$$\begin{aligned} f(g(x)) &= \frac{1}{4}(\sqrt[3]{4x-12})^3 - 3 \\ &= \frac{1}{4}(4x-12) - 3 \\ &= x - 3 - 3 \\ &= x - 6 \end{aligned}$$

No, not inverses

Find the inverse of each function. Graph both functions and verify the inverse relationship on the graph.

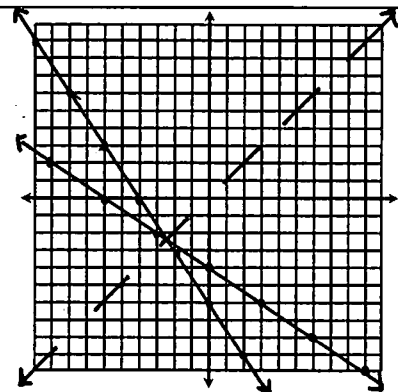
14. $f(x) = -\frac{2}{3}x - 4$

$$x = -\frac{3}{2}y - 4$$

$$y = -\frac{3}{2}x - 4$$

$$-\frac{3}{2}x = y + 4$$

$$f^{-1}(x) = -\frac{3}{2}x - 6$$



15. $f(x) = -x^2 + 6$ (if $x \geq 0$)

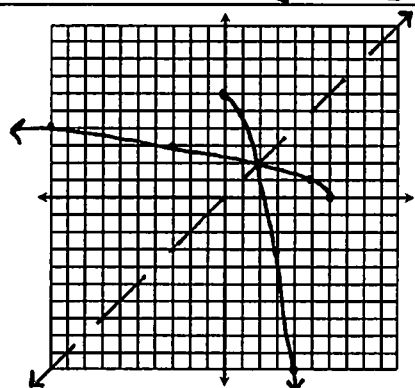
$$y = -x^2 + 6$$

$$-6 = -y^2$$

$$-x + 6 = y^2$$

$$\sqrt{-x+6} = y$$

$$\begin{aligned} f^{-1}(x) &= \sqrt{-x+6} \\ x &\leq 6 \end{aligned}$$



Unit 6 Test Study Guide (Radical Functions)

Name: _____

Date: _____ Per: _____

Topic 1: Simplifying Radicals

Perfect Squares: 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225, 256, 289, 324, 361, 400, 441, 484, 529, 576, 625, 676, 729, 784, 841, 900, 961, 1024, 1089, 1156, 1225, 1296, 1369, 1444, 1521, 1600, 1681, 1764, 1849, 1936, 2025, 2116, 2209, 2304, 2401, 2500, 2601, 2704, 2809, 2916, 3025, 3136, 3249, 3364, 3481, 3600, 3721, 3844, 3969, 4096, 4225, 4356, 4489, 4624, 4761, 4900, 5041, 5184, 5329, 5476, 5625, 5776, 5929, 6084, 6241, 6400, 6561, 6724, 6891, 7064, 7241, 7424, 7611, 7804, 7999, 8196, 8399, 8596, 8799, 8996, 9199, 9396, 9599, 9796, 9999

Perfect Cubes: 1, 8, 27, 64, 125, 216, 343, 512, 729, 1000, 1331, 1728, 2197, 2744, 3375, 4096, 4913, 5832, 6859, 8000, 9268, 10648, 12167, 13868, 15768, 17869, 20180, 22711, 25480, 28497, 31772, 35325, 39176, 43336, 47817, 52630, 57797, 63340, 69281, 75632, 82405, 89612, 97265, 105388, 113993, 123104, 132735, 142900, 153613, 164888, 176739, 189180, 202225, 215880, 230160, 245080, 260660, 276920, 293875, 311540, 330032, 349376, 369592, 390700, 412720, 435672, 459584, 484475, 510376, 537308, 565300, 594384, 624584, 655928, 688544, 722464, 757720, 794344, 832368, 871824, 912744, 955160, 999104, 1044608, 1091704, 1140432, 1190824, 1242912, 1296736, 1352328, 1409720, 1468960, 1529984, 1592832, 1657544, 1724160, 1792720, 1863264, 1935832, 2010464, 2087200, 2166080, 2247152, 2330464, 2416064, 2504000, 2594320, 2687088, 2782360, 2880192, 2980544, 3083480, 3189064, 3297360, 3408432, 3522344, 3639160, 3758944, 3881760, 4007680, 4135760, 4267064, 4401664, 4539632, 4681040, 4825960, 4974464, 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13. $\frac{5\sqrt{6}}{\sqrt{15}} = \frac{5\sqrt{2} \cdot \sqrt{3}}{\sqrt{5} \cdot \sqrt{3}}$ $= \frac{5\sqrt{2}}{\sqrt{5}}$ $= \frac{5}{\sqrt{10}}$	14. $\sqrt{\frac{10m^3}{18m}} = \sqrt{\frac{5m^2}{9}}$ $= \frac{m\sqrt{5}}{3}$	15. $\frac{(\sqrt{5}-\sqrt{2})\sqrt{2}}{(4\sqrt{2})\sqrt{2}}$ $= \frac{\sqrt{10}-\sqrt{4}}{4\sqrt{4}}$ $= \frac{\sqrt{10}-2}{8}$
16. $\frac{(2)(5-3\sqrt{2})}{(5+3\sqrt{2})(5-3\sqrt{2})}$ $\frac{10-6\sqrt{2}}{25-15\sqrt{2}+15\sqrt{2}-9\sqrt{4}}$ $= \frac{10-6\sqrt{2}}{25-18} = \frac{10-6\sqrt{2}}{7}$	17. $\frac{(2+\sqrt{7})(5+\sqrt{7})}{(5-\sqrt{7})(5+\sqrt{7})}$ $\frac{10+2\sqrt{7}+5\sqrt{7}+7}{25+5\sqrt{7}-5\sqrt{7}-\sqrt{49}}$ $= \frac{10+7\sqrt{7}}{25-7} = \frac{17+7\sqrt{7}}{18}$	

Topic 3: Rational Exponents

Rewrite in radical form. Simplify if possible.

18. $16^{\frac{1}{4}} = \sqrt[4]{16}$ $= 2$	19. $a^{\frac{2}{3}} = \sqrt[3]{a^2}$	20. $(2y)^{\frac{5}{2}} = \sqrt[2]{(2y)^5}$ $= \sqrt[2]{32y^5}$ $= \sqrt{16y^4} \sqrt{2y} = 4y^2\sqrt{2y}$	21. $18m^{\frac{7}{4}}$ $18\sqrt[4]{m^7}$ $= 18\sqrt[4]{m^4} \sqrt[4]{m^3}$ $= 18m\sqrt[4]{m^3}$
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Rewrite in exponential form.

22. $\sqrt[3]{17m^2}$ $(17m^2)^{\frac{1}{3}}$ $= 17^{\frac{1}{3}} m^{\frac{2}{3}}$	23. $(34)^{\frac{3}{4}}$ $= 34^{\frac{3}{4}}$ $= 34^{\frac{3}{4}}$	24. $\sqrt{12a^9b^8}$ $(12a^9b^8)^{\frac{1}{2}}$ $= 12^{\frac{1}{2}} a^{\frac{9}{2}} b^4$	25. $\sqrt[6]{4r^2s^{10}}$ $(2^2r^2s^{10})^{\frac{1}{6}}$ $= 2^{\frac{1}{3}} r^{\frac{1}{3}} s^{\frac{5}{3}}$
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Simplify each expression. Write your final answers in simplest radical form.

26. $x^{\frac{5}{4}}$ $= x^{\frac{5}{4}}$ $= x^{\frac{5}{4}}$ $= \sqrt[4]{x^5}$	27. $(81^8)^{\frac{1}{4}} = 81^{-\frac{8}{4}}$ $= 81^{-2}$ $= \frac{1}{6561}$	28. $\frac{k^{\frac{13}{3}}}{k} = k^{10/3}$ $= \sqrt[3]{k^{10}}$ $= \sqrt[3]{k^9} \sqrt[3]{k} = k^3 \sqrt[3]{k}$
29. $\sqrt[4]{m^{16}} \cdot \sqrt[4]{m^3}$ $m^{\frac{16}{4}} \cdot m^{\frac{3}{4}} = m^{19/4}$ $= \sqrt[4]{m^{16}} \sqrt[4]{m^3}$ $= m^4 \sqrt[4]{m^3}$	30. $\frac{2^3}{\sqrt{2^7}} = \frac{2^3}{2^{7/2}}$ $= 2^{-1/2}$ $= \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$	31. $\frac{\sqrt{x^3} \cdot \sqrt[4]{x^3}}{x} = \frac{x^{\frac{3}{2}} \cdot x^{\frac{3}{4}}}{x}$ $= \frac{x^{\frac{9}{4}}}{x}$ $= x^{\frac{5}{4}} = x \sqrt[4]{x}$

Topic 4: Solving Radical Equations

Solve each equation. Check for extraneous solutions.

32. $\sqrt{5x-1}+4=11$
 $(\sqrt{5x-1})^2=(7)^2$
 $5x-1=49$
 $5x=50$
 $x=10$

ck:
 $\sqrt{5 \cdot 10 - 1} + 4 = 11$
 $\sqrt{49} + 4 = 11$
 $7 + 4 = 11 \checkmark$

33. $1-2\sqrt{m+12}=5$
 $-2\sqrt{m+12}=4$
 $(\sqrt{m+12})^2=(-2)^2$
 $m+12=4$
 $m=-8$

ck:
 $1-2\sqrt{-8+12}=5$
 $1-2\sqrt{4}=5$
 $1-2(2)=5$
 $1-4=5$
 $-3=5$
 Extraneous

34. $((9a+45)^{\frac{1}{3}})^3=(3)^3$
 $9a+45=27$
 $9a=-18$
 $a=-2$

ck:
 $(9(-2)+45)^{\frac{1}{3}}=3$
 $(-18+45)^{\frac{1}{3}}=3$
 $27^{\frac{1}{3}}=3$
 $3=3 \checkmark$

35. $(\sqrt{3n-27})^2=(3)^2$
 $3n-27=9$
 $3n=36$
 $n=12$

ck:
 $\sqrt{3 \cdot 12 - 27} = \sqrt{43 - 27}$
 $\sqrt{16} = \sqrt{16}$
 $4 = 4 \checkmark$

36. $((18+7k)^{\frac{1}{2}})^2=(k)^2$
 $18+7k=k^2$
 $0=k^2-7k-18$
 $0=(k-9)(k+2)$
 $k=9$ or $k=-2$
 $k=9$ or $k=-2$

ck:
 $(18+7(9))^{\frac{1}{2}}=9$
 $(72+63)^{\frac{1}{2}}=9$
 $(135)^{\frac{1}{2}}=9$
 $11.618 \neq 9$
 Extraneous
 $(18+7(-2))^{\frac{1}{2}}=-2$
 $(-6)^{\frac{1}{2}}=-2$
 $(4)^{\frac{1}{2}} \neq -2$
 Extraneous

$(\sqrt{6x+19}-4)^2=x^2$
 $6x+19=x^2+8x+16$
 $0=x^2+2x-3$
 $0=(x+3)(x-1)$
 $x=-3$ or $x=1$

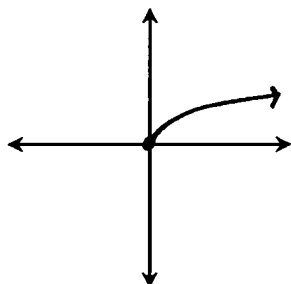
ck:
 $\sqrt{6(-3)+19}-4=-3$
 $\sqrt{1}-4=-3$
 $1-4=-3 \checkmark$
 $\sqrt{6(1)+19}-4=1$
 $\sqrt{25}-4=1$
 $5-4=1 \checkmark$

Topic 5: Graphing Radical Functions

Identify the parent function and sketch the shape of the graph.

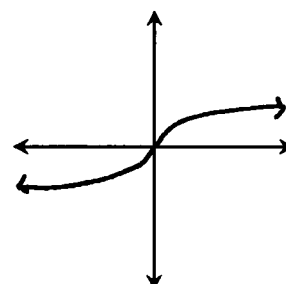
SQUARE ROOT

$f(x)=\sqrt{x}$



CUBE ROOT

$f(x)=\sqrt[3]{x}$



Describe the transformations on each function compared to its parent function.

38. $f(x) = \frac{3}{4}\sqrt{x} + 1$ Vertical comp.
by $\frac{3}{4}$,
up 1

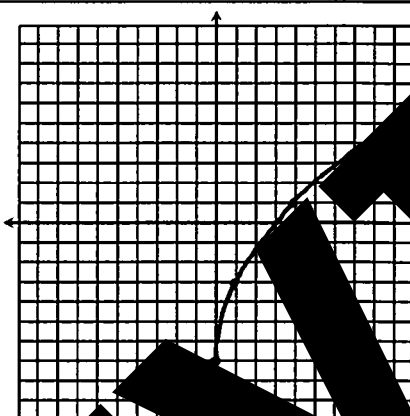
39. $f(x) = 2\sqrt[3]{x+5} - 8$
Vert. stretch by 2,
left 5, down 8

40. The cube root parent function is reflected about the x -axis, then shifted so that the point of inflection is located at $(-6, -2)$. Write an equation that represents this new function.

$$f(x) = -\sqrt[3]{x+6} - 2$$

Graph each function and identify its key characteristics.

41. $f(x) = 4\sqrt{x} - 7$



D: $\{x | x \geq 0\}$ R: $\{y | y \geq -7\}$

Endpoint: $(0, -7)$

End Behavior:

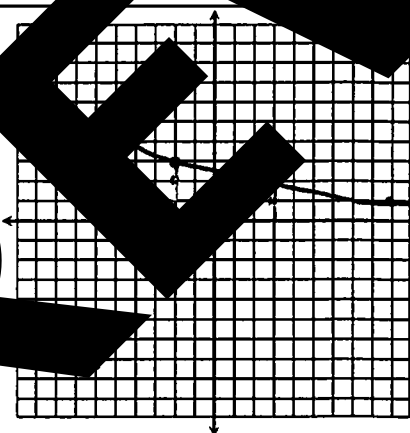
As $x \rightarrow \infty$, $f(x) \rightarrow \infty$

As $x \rightarrow 0$, $f(x) \rightarrow -7$

Increasing Interval(s): $[0, \infty)$

Decreasing Interval(s): N/A

42. $f(x) = -\sqrt{x+6} + 5$



D: $\{x | x \geq -6\}$ R: $\{y | y \leq 5\}$

Endpoint: $(-6, 5)$

End Behavior:

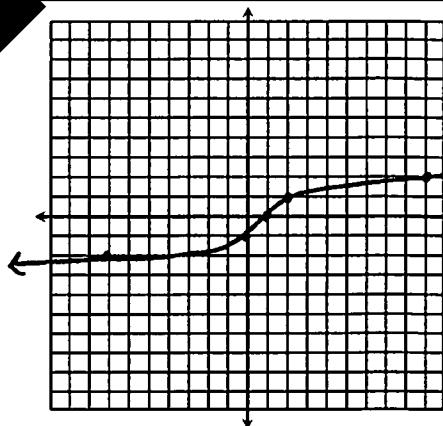
As $x \rightarrow \infty$, $f(x) \rightarrow -\infty$

As $x \rightarrow -6$, $f(x) \rightarrow 5$

Increasing Interval(s): N/A

Decreasing Interval(s): $[-6, \infty)$

43. $f(x) = \frac{1}{x-1}$



D: $\mathbb{R}$ R: $\mathbb{R}$

Point of Inflection: $(1, 0)$

End Behavior:

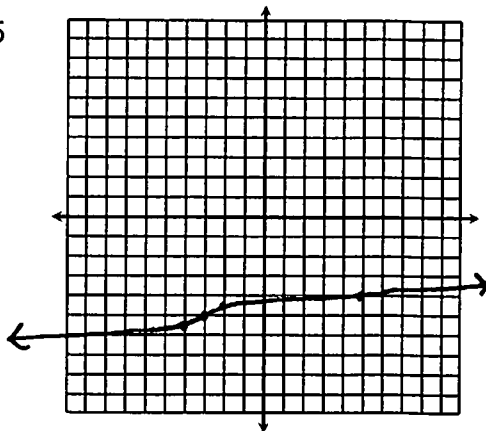
As $x \rightarrow \infty$, $f(x) \rightarrow 0$

As $x \rightarrow -\infty$, $f(x) \rightarrow 0$

Increasing Interval(s): $(-\infty, \infty)$

Decreasing Interval(s): N/A

44. $f(x) = \frac{1}{2}\sqrt[3]{x+3} - 5$



D: $\mathbb{R}$ R: $\mathbb{R}$

Point of Inflection: $(-3, -5)$

End Behavior:

As $x \rightarrow \infty$, $f(x) \rightarrow \infty$

As $x \rightarrow -\infty$, $f(x) \rightarrow -\infty$

Increasing Intervals: $(-\infty, \infty)$

Decreasing Intervals: $\emptyset$

Topic 6: Inverse Functions

Find the inverse of each relation. Determine if it is a one-to-one function.

45. $R = \{(5, 1), (-2, -3), (5, 9), (2, 7)\}$

$R^{-1} = \{(1, 5), (-3, -2), (9, 5), (7, 2)\}$

No; Not one-to-one

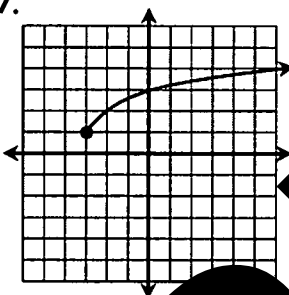
46. $R = \{(-2, 4), (-4, 4), (5, 5)\}$

$R^{-1} = \{(4, -2), (4, -4), (5, 5)\}$

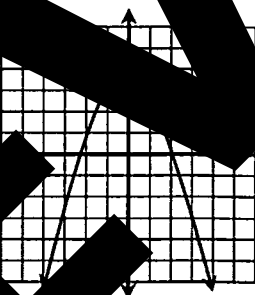
No; Not one-to-one

Determine whether the relations below represent one-to-one functions.

47.

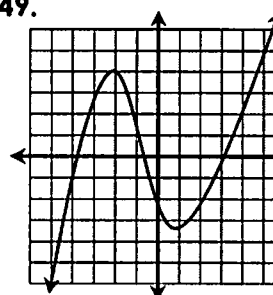


48.



No

49.

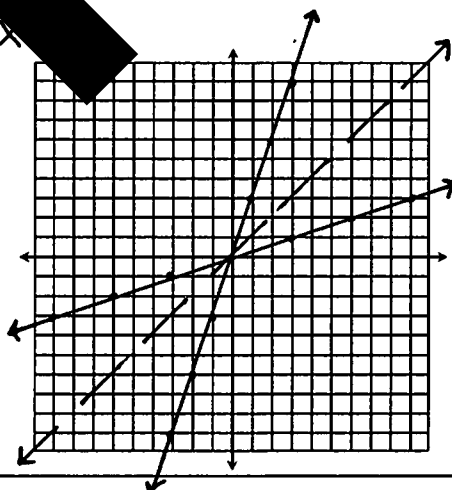


No

Find the inverse of each function. Verify the inverse relationship on the graph.

50. $f(x) = \frac{1}{3}x - 7$

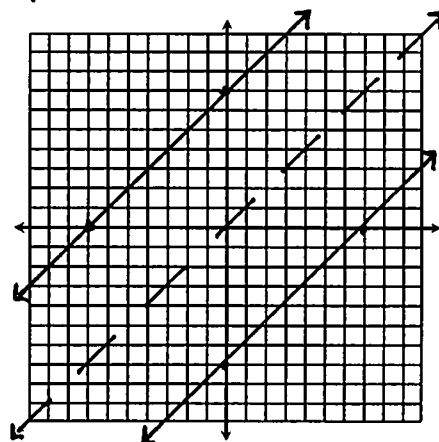
$f^{-1}(x) = 3x + 21$



51. $f(x) = x + 7$

$x - 7 = y$

$f^{-1}(x) = x - 7$



$$52. f(x) = \frac{2}{5}x - 1 \rightarrow x = \frac{2}{5}y - 1$$

$$x + 1 = \frac{2}{5}y$$

$$\frac{5}{2}x + \frac{5}{2} = y$$

$$f^{-1}(x) = \frac{5}{2}x + \frac{5}{2}$$

$$53. f(x) = \sqrt{x} + 3 \rightarrow x = \sqrt{y} + 3$$

$$x - 3 = \sqrt{y}$$

$$(x - 3)^2 = y$$

$$f^{-1}(x) = (x - 3)^2, x \geq 3$$

Determine whether the pair of functions are inverse functions.

$$54. f(x) = 8x - 12 \text{ and } g(x) = \frac{1}{8}x + \frac{3}{2}$$

$$f(g(x)) = 8\left(\frac{1}{8}x + \frac{3}{2}\right) - 12$$

$$= x + 12 - 12$$

$$= x$$

$$g(f(x)) = \frac{1}{8}(8x - 12) + \frac{3}{2}$$

$$= x - \frac{3}{2} + \frac{3}{2}$$

$$= x$$

Yes; inverses

$$55. f(x) = \frac{1}{9}x + 1 \text{ and } g(x) = 9x + 9$$

$$f(g(x)) = \frac{1}{9}(9x + 9) + 1$$

$$= x + 1 + 1$$

$$= x + 2$$

Not inverses

$$56. f(x) = \frac{1}{2}x^3 - 2 \text{ and } g(x) = \sqrt[3]{2x + 4}$$

$$f(g(x)) = \frac{1}{2}(\sqrt[3]{2x + 4})^3 - 2$$

$$= \frac{1}{2}(2x + 4) - 2$$

$$= x + 2 - 2$$

$$= x$$

$$g(f(x)) = \sqrt[3]{2\left(\frac{1}{2}x^3 - 2\right) + 4}$$

$$= \sqrt[3]{x^3 - 4 + 4}$$

$$= \sqrt[3]{x^3}$$

$$= x$$

Yes; inverses

Name: _____

Date: _____ Per: _____

Unit 6 Test
Radical Functions

1. Simplify the expression below.

$$\frac{\sqrt[3]{81m^6n^4}}{\sqrt[3]{27m^6n^3} \sqrt[3]{3m^2n}}$$

- A. $9m^2n^3\sqrt{n}$
- B. $9m^3n^3\sqrt{m^2n}$
- C. $3m^2n^3\sqrt{3n}$
- D. $3m^2n^3\sqrt{3m^2n}$

D

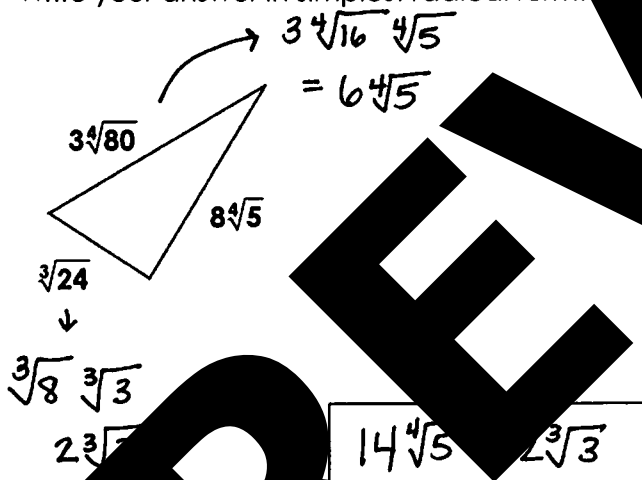
2. Simplify the expression below.

$$\frac{5xy\sqrt{28x} + \sqrt{63x^3y^2}}{5xy\sqrt{4}\sqrt{7x}}$$

- A. $13xy\sqrt{7x}$
- B. $13x^2$
- C. $29xy\sqrt{7x}$
- D. $29x^2y^2\sqrt{7x}$

A

3. Find the perimeter of the triangle below.
Write your answer in simplest radical form.



4. Simplify the expression below.

$$\sqrt[4]{8w^3} \cdot 3\sqrt[4]{32w^5}$$

- A. $24w^2$
- B. $96w^2$
- C. $24w^3\sqrt[4]{w^3}$
- D. $96w^3\sqrt[4]{w^3}$

A

5. Simplify

$$\begin{aligned} & \sqrt{18} - \sqrt{3} - 2\sqrt{3} \\ & 2\sqrt{18} + \sqrt{108} - 2\sqrt{3} \\ & 2\sqrt{9}\sqrt{2} + \sqrt{36}\sqrt{3} - 2\sqrt{3} \\ & 6\sqrt{2} + 6\sqrt{3} - 2\sqrt{3} \end{aligned}$$

$$6\sqrt{2} + 4\sqrt{3}$$

6. Simplify the expression below.

$$\begin{aligned} & (4\sqrt{3} - 1)(\sqrt{3} - 6) \\ & 4\sqrt{9} - 24\sqrt{3} - \sqrt{3} + 6 \\ & 12 - 25\sqrt{3} + 6 \end{aligned}$$

$$18 - 25\sqrt{3}$$

7. Simplify the expression below.

$$(2\sqrt{10} - \sqrt{6})(2\sqrt{10} + \sqrt{6})$$

$$4\sqrt{100} + 2\sqrt{60} - 2\sqrt{60} - \sqrt{36}$$

$$4 \cdot 10 - 6$$

34

8. The side length of a square is $(\sqrt{2} - 3\sqrt{5})$ meters. Find the area of the square and write your answer in simplest radical form.

$$(\sqrt{2} - 3\sqrt{5})(\sqrt{2} - 3\sqrt{5})$$

$$\sqrt{4} - 3\sqrt{10} - 3\sqrt{10} + 9\sqrt{25}$$

$$2 - 6\sqrt{10} + 45$$

47 - 6\sqrt{10}

9. Simplify the expression below.

$$\frac{\sqrt{294k^{30}}}{\sqrt{3k^5}} = \sqrt{98k^{25}}$$

$$= \sqrt{49k^{24}} \sqrt{2k}$$

- A. $7k^2\sqrt{2}$
 B. $7k^3\sqrt{2}$
 C. $7k^5\sqrt{2}$
 D. $7k^{12}\sqrt{2k}$

D

10. Simplify the expression below.

$$\frac{\sqrt{-375}}{\sqrt{-10}} = \frac{\sqrt{2^3 \cdot 3 \cdot 5}}{\sqrt{2 \cdot 5}} = \frac{2^{\frac{3}{2}} \cdot \sqrt{3}}{2^{\frac{1}{2}} \cdot \sqrt{5}} = \frac{2^{\frac{3}{2}-\frac{1}{2}} \cdot \sqrt{3}}{\sqrt{5}} = \frac{2^1 \cdot \sqrt{3}}{\sqrt{5}} = \frac{2\sqrt{3}}{\sqrt{5}}$$

B. $-\frac{10}{3}$ D. $-\frac{1}{10}$

D

11. Simplify the expression below.

$$\frac{12\sqrt{3}}{\sqrt{8}}$$

$$\frac{12\sqrt{24}}{8\sqrt{6}}$$

$$\frac{12\sqrt{24}}{8\sqrt{6}} = \frac{12\sqrt{4 \cdot 6}}{8\sqrt{6}} = \frac{12 \cdot 2\sqrt{6}}{8\sqrt{6}} = \frac{24\sqrt{6}}{8\sqrt{6}} = 3$$

3\sqrt{6}

12. Simplify the expression below.

$$\frac{\sqrt{5x}}{\sqrt{12}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{15x}}{\sqrt{36}}$$

- A. $\frac{\sqrt{5x}}{6}$ C. $4\sqrt{15x}$
 B. $\frac{\sqrt{15x}}{6}$ D. $\frac{\sqrt{15x}}{3}$

B

13. Simplify the expression below.

$$\frac{(4\sqrt{5} + 4)\sqrt{2}}{\sqrt{2} \cdot \sqrt{2}}$$

$$\frac{\sqrt{10} + 4\sqrt{2}}{2}$$

2\sqrt{10} + 2\sqrt{2}

14. Simplify the expression below.

$$\frac{(3+5\sqrt{7})(3+\sqrt{7})}{(3-\sqrt{7})(3+\sqrt{7})}$$

$$\frac{9+3\sqrt{7}+15\sqrt{7}+5\sqrt{49}}{9+3\sqrt{7}-3\sqrt{7}-\sqrt{49}}$$

$$= \frac{9+18\sqrt{7}+35}{9-7}$$

$$= \frac{44+18\sqrt{7}}{2}$$

22+9\sqrt{7}

15. Which is equivalent to the expression below?

$$\sqrt{16x^7y^4}$$

$$4x^{7/2}y^{4/2}$$

A. $4x^{7/2}y^{1/2}$

C. $8x^{7/2}y^{1/2}$

B. $4x^{7/2}y^2$

D. $8x^{7/2}y^2$

B

16. Which is equivalent to the expression below?

$$a^{5/4}b^{13/4}$$

$$a \cdot a^{1/4} \cdot b^3 \cdot b^{1/4}$$

A. $\frac{1}{4}\sqrt{a^5b^{13}}$

C. a^2b^8

B. $a^4b^4\sqrt{ab}$

D. $ab^3\sqrt{ab}$

D

17. Simplify the expression below.

$$(2x)^{1/3} \cdot (2x)^{4/3}$$

A. $2x^3\sqrt{4x^2}$

B. $2x^3\sqrt{2x}$

C. $4x^3\sqrt{4x^2}$

D. $4x^3\sqrt{2x}$

$$(2x)^{5/3}$$

$$(2x) \cdot (2x)^{2/3}$$

A

18. Simplify the expression below.

$$\frac{9^{7/4}}{9^{9/4}} = 9^{-2/4}$$

$$= 9^{-1/2}$$

$$= \frac{1}{\sqrt{9}}$$

$\frac{1}{3}$

19. Simplify the expression below.

$$\left(\frac{3}{p^2}\right)^{7/3} = p^{21/3}$$

$$= p^7$$

A. p^5

B. $p^2\sqrt{p}$

C. $4\sqrt{p^{20}}$

D. $p^{10}\sqrt{p}$

C

20. Simplify the expression below.

$$\frac{(-54)^2}{\sqrt[3]{(-54)^5}}$$

$$\frac{(-54)^2}{(-54)^{5/3}} = (-54)^{2-5/3} = \sqrt[3]{-27} \cdot \sqrt[3]{2}$$

$-3\sqrt[3]{2}$

21. Solve the equation below.

$$\sqrt{6n-5}+1=8$$

$$\sqrt{6n-5}=7$$

$$6n-5=49$$

$$6n=54$$

$$n=9$$

ck:

$$\sqrt{6 \cdot 9 - 5} + 1 = 8$$

$$\sqrt{49} + 1 = 8$$

$$7 + 1 = 8 \checkmark$$

$n=9$

22. Solve the equation below.

$$-5\sqrt[3]{2x+3} = -15$$

$$\sqrt[3]{2x+3} = 3$$

$$2x+3=27$$

$$2x=24$$

$$x=12$$

ck:

$$-5\sqrt[3]{2 \cdot 12 + 3} = -15$$

$$-5\sqrt[3]{27} = -15$$

$$-5 \cdot 3 = -15 \checkmark$$

$x=12$

23. Solve the equation below.

$$\begin{aligned} 7 - (3k - 29)^{\frac{1}{4}} &= 5 \\ -(3k - 29)^{\frac{1}{4}} &= -2 \\ (3k - 29)^{\frac{1}{4}} &= 2 \\ 3k - 29 &= 16 \\ 3k &= 45 \\ k &= 15 \end{aligned}$$

ck:

$$\begin{aligned} 7 - (3 \cdot 15 - 29)^{\frac{1}{4}} &= 5 \\ 7 - (45 - 29)^{\frac{1}{4}} &= 5 \\ 7 - 2 &= 5 \checkmark \end{aligned}$$

$$k = 15$$

24. Solve the equation below.

$$\begin{aligned} (7 - 2c)^{\frac{1}{2}} &= (1 - 5c)^{\frac{1}{2}} \\ 7 - 2c &= 1 - 5c \\ 6 - 2c &= -5c \\ 6 &= -3c \\ -2 &= c \end{aligned}$$

ck:

$$\begin{aligned} (7 - 2(-2))^{\frac{1}{2}} &= (1 - 5(-2))^{\frac{1}{2}} \\ (7 + 4)^{\frac{1}{2}} &= (1 + 10)^{\frac{1}{2}} \\ 11^{\frac{1}{2}} &= 11^{\frac{1}{2}} \end{aligned}$$

25. Solve the equation below.

$$\begin{aligned} n &= \sqrt{13n - 40} \\ n^2 &= 13n - 40 \\ n^2 - 13n + 40 &= 0 \\ (n - 8)(n - 5) &= 0 \\ n = 8 & \quad n = 5 \end{aligned}$$

ck:

$$\begin{aligned} 8 &= \sqrt{13 \cdot 8 - 40} \quad | \quad 5 = \sqrt{13 \cdot 5 - 40} \\ 8 &= \sqrt{64} \checkmark \quad | \quad 5 = \sqrt{25} \checkmark \end{aligned}$$

$$n = 8$$

26. Solve the equation below.

$$\begin{aligned} y + 1 &= y \\ y + 15 &= y \\ 0 &= y^2 + 8y + 16 \\ 0 &= (y + 4)^2 \\ 0 &= (y + 1)(y + 1) \\ y &= -1 \quad | \quad y = -1 \end{aligned}$$

$$\begin{aligned} \sqrt{6(-1) + 15} - 4 &= -1 \\ \sqrt{-6 + 15} - 4 &= -1 \\ 3 - 4 &= -1 \checkmark \end{aligned}$$

$$y = -1$$

27. The square root parent function is transformed so that the endpoint is located at $(-7, 1)$. Write an equation that represents this function.

$$f(x) = \sqrt{x + 7} + 1$$

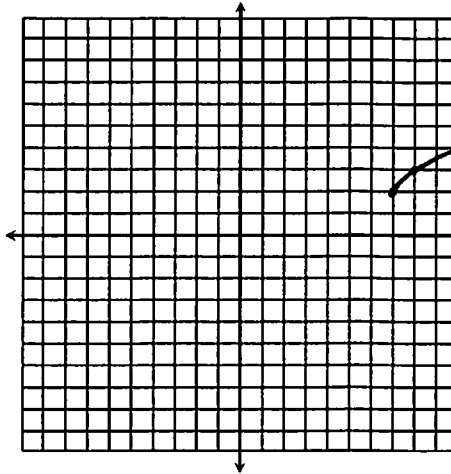
28. The transformations that take place by graphing the function below in respect to its parent function are: Check all that apply.

$$f(x) = -\frac{2}{3}\sqrt{x + 9} - 4$$

☐ Up 9 units	☐ Up 4 units	☐ Vertical Stretch
☐ Down 9 units	☒ Down 4 units	☒ Vertical Compression
☒ Left 9 units	☐ Left 4 units	☒ Reflection in x-axis
☐ Right 9 units	☐ Right 4 units	☐ Reflection in y-axis

Graph each function and identify its key characteristics.

29. $f(x) = \sqrt{x-7} + 2$



D: $\{x | x \geq 7\}$ R: $\{y | y \geq 2\}$

Endpoint: $(7, 2)$

End Behavior:

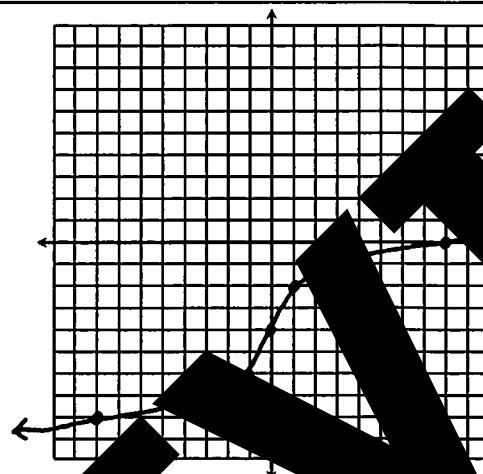
As $x \rightarrow \infty$, $f(x) \rightarrow \infty$

As $x \rightarrow 7$, $f(x) \rightarrow 2$

Increasing Interval(s): $[7, \infty)$

Decreasing Interval(s): N/A

30. $f(x) = 2\sqrt[3]{x} - 4$



D: $\mathbb{R}$ R: $\mathbb{R}$

Point of Inflection: $(0, -4)$

End Behavior:

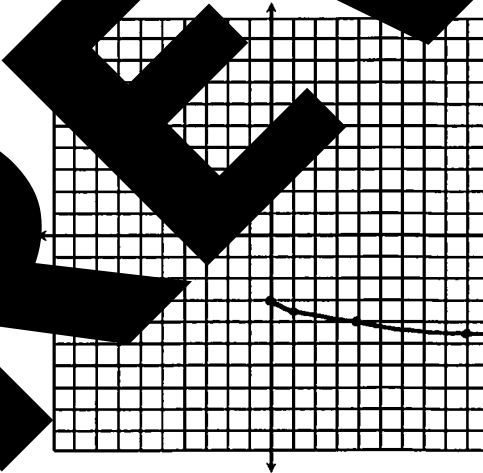
As $x \rightarrow \infty$, $f(x) \rightarrow \infty$

As $x \rightarrow -\infty$, $f(x) \rightarrow -\infty$

Increasing Interval(s): $(-\infty, \infty)$

Decreasing Interval(s): N/A

31. $f(x) = -\frac{1}{2}\sqrt{x} - 3$



D: $\{x | x \geq 0\}$ R: $\{y | y \leq -3\}$

Endpoint: $(0, -3)$

End Behavior:

As $x \rightarrow \infty$, $f(x) \rightarrow -\infty$

As $x \rightarrow 0$, $f(x) \rightarrow -3$

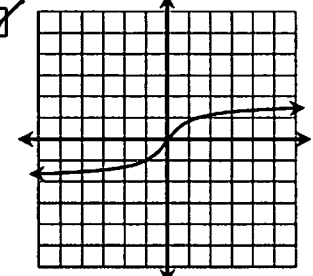
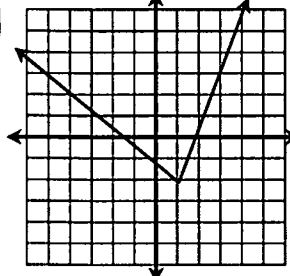
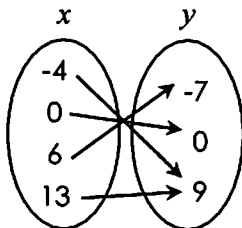
Increasing Interval(s): N/A

Decreasing Interval(s): $[0, \infty)$

Which of the following relations represent a one-to-one function? Check all that apply.



x	y
-2	-10
-1	-5
1	5
2	10



33. Write the inverse of the function below. Graph both functions to verify the relationship.

$$f(x) = -\frac{1}{4}x - 2$$

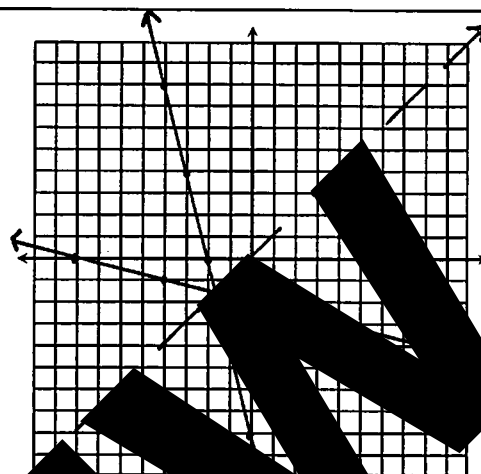
$$x = -\frac{1}{4}y - 2$$

$$x + 2 = -\frac{1}{4}y$$

$$-4x - 8 = y$$

Inverse Function:

$$f^{-1}(x) = -4x - 8$$



Determine whether the functions below are inverse functions. Prove your answer algebraically.

34. $f(x) = \frac{1}{3}x - 3$ and $g(x) = 6x + 18$

$$f(g(x)) = \frac{1}{3}(6x + 18) - 3$$

$$= 2x + 6 - 3$$

$$= 2x + 3$$

☐ Yes

☒ No

35. $f(x) = \sqrt[4]{2x-6}$ and $g(x) = \frac{1}{2}x + 3$

$$f(g(x)) = \sqrt[4]{2(\frac{1}{2}x + 3) - 6}$$

$$= \sqrt[4]{x + 6 - 6}$$

$$= \sqrt[4]{x}$$

$$g(f(x)) = \frac{1}{2}(\sqrt[4]{2x-6}) + 3$$

$$= \frac{1}{2}(2x-6) + 3$$

$$= x - 3 + 3$$

$$= x \checkmark$$

☒ Yes

☐ No

BONUS: Solve $8x^3 - 12x^2 + 6x - 1 = 0$

$$8x^3 - 12x^2 + 6x - 1 = (2x-1)(4x^2-4x+1)$$

$$= (2x-1)(4x^2-4x+1)$$

$$= 8x^3 - 8x^2 + 2x - 4x^2 + 4x - 1$$

$$8x^3 - 12x^2 + 6x - 1$$

$$0 = -12x^2 + 6x$$

$$0 = -6x(2x-1)$$

$$x=0 \quad x=\frac{1}{2}$$

$$\text{ck: } \sqrt[3]{8(0)^3-1} + 1 = 2(0)$$

$$\sqrt[3]{-1} + 1 = 0 \checkmark$$

$$\sqrt[3]{8(\frac{1}{2})^3-1} + 1 = 2(\frac{1}{2})$$

$$\sqrt[3]{1-1} + 1 = 1$$

$$\sqrt[3]{1-1} + 1 = 1$$

$$1 = 1 \checkmark$$

$$x=0, x=\frac{1}{2}$$

CREDITS

I use clipart and
fonts in my products



Art with Penny



Many thanks to these
talented artists!